Fundamentals of mathematical logic ESP. Book 1

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P R O L O G U E

"Para aquellos que no conocen las matemáticas, es difícil sentir la belleza de la naturaleza. Si quieres apreciarla, es necesario aprender el lenguaje en el que habla"
Richard Feynman

It is an honor and an undeniable privilege to have the opportunity to preface this work that is presented to you, distinguished readers, as tangible evidence of the remarkable intellectual effort of a group of professors of the Escuela Superior Politecnica de Chimborazo (ESPOCH). The commitment inherent in the writing of this book is magnified by the deep gratitude -to my colleagues and the university- for the invaluable responsibility of letting me perambulate the linguistic-mathematical compendium that you have in front of you at this moment. As an academic and professional, I am compelled to praise the uniqueness of this project, whose relevance transcends the pages that comprise it, and which I hope you will appreciate in its proper measure.

The essential need to acquire sufficient English language skills has become a condition sine qua non in the current context of globalization and interconnected markets. While modernity has brought countless changes in human dynamics, it has also brought with it inexcusable demands such as the mastery of this language, which is no longer a mere subsidiary skill, but rather a requirement for the correct occupational, professional and intellectual immersion in an increasingly interdependent and competitive world. In spite of having detractors, who are fundamentally ideological, English continues to be in the 21st century a useful tool capable of interconnecting innumerable processes in dissimilar latitudes, under a standard code and universally accepted as valid.

In this sense, the understanding of such an important language opens a range of opportunities and projections that are not only subject to the simple understanding of a culture or the establishment of communication channels, but also facilitates the integration of work environments characterized by multiculturalism and where the interaction and/or dynamics revolves around this language. Therefore, the lack of English proficiency implies a continuous attack against the ability to establish meaningful and value-added relationships at a global level, limiting opportunities for professional and intellectual growth, as well as the expansion of networks of influence.

Furthermore, on the threshold of this linguistic compendium, which is intended for avid learners of the mathematical discipline, we embark on a pedagogical journey that transcends the traditional boundaries of teaching, given that the language taught at university is generally always intended for the introduction of instrumental knowledge, which in many cases lacks depth and adaptability to science. Therefore, the synergy between the Anglo-Saxon language and the science of numbers in this work doubly promotes fluency, constituting an ideal catalyst for the forging of essential skills required by contemporaneity and also by the science that gathers us here.

It is worth mentioning that the complex problems faced by societies around the globe challenge science and its ability to provide timely responses, not only because of the whirlwind of changes arising from current systemic structures, but also because of their transience in the timeline. This makes the approach to these solutions to be subsumed in a transdisciplinary perspective that guarantees the transcendence of the individual niches of each scientific discipline for a true commonwealth; in other words, the collaboration between scientists, engineers, humanists and professionals from different areas is de facto necessary to articulate comprehensive answers that consider the social, ethical-philosophical and technical dimensions of these multifaceted phenomena.

Now, the truth is that this colossal intellectual work could generate - a priori - within the university community, certain doubts regarding the reliability of the mathematical and linguistic contents immersed in the work, which seek to forge a professional capable of developing fluently in both areas; However, it is necessary to argue in defense of the text and its authors, that it has been reviewed on countless occasions by professionals who make up a multidisciplinary group, which give faithful testimony that it is not only in order according to the treatment of content and pedagogy used, but also to the relevance of this as a teaching tool for the training of the new generation of professionals in mathematics.

The truth is that for the mathematics students of the Escuela Superior Politécnica de Chimborazo (ESPOCH), this book represents the articulated and joint work of scientific disciplines that at first glance may not have any relationship, giving a clear demonstration that the approach to science can occur from different approaches and perspectives, only if the final direction is only one. Regardless of the processes that led to its realization and the time it took for its elaboration, this book will contribute to strengthen knowledge in English, and in parallel in mathematical logic as an avant-garde university strategy, oriented to form comprehensive professionals who are in tune with their science of study, and also with the up-coming changes that our societies are experiencing in post-modern times.

Thus, immersing oneself in the following pages not only represents an act of learning, but also an immersion in the exciting intersection between language and numbers, where expository clarity and conceptual depth go hand in hand to guide our distinguished readers towards a synergistic mastery of both disciplines. The value of this idea lies in allowing science to be effectively linked with other areas of knowledge without the intention of superimposing one over the other, but rather as a strategy of positive complementarity that will end up benefiting those who are currently training to become the next generation of successors.

I wish them success in their journey through the sixteen lessons that comprise it, and I commend the role of the authors, who against all odds today materialize their idea of producing a text that will be the ideal channel for mathematics students to exponentially increase their knowledge in two areas together. Finally, this work represents a clear challenge against the traditional frontiers of knowledge and adheres to a more integrative approach that allows unveiling a more complete and revealing educational universe, adapted to our new socio-educational realities.

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Introduction

On the one hand, in a world that is increasingly interconnected, especially due to the exponential technological advances that are presented and are necessary on a daily basis, English is a language that takes ground in everyday life and becomes more common and essential to participate in this dynamic, as well as to facilitate connectivity and interactivity between multiple people through oral and/or written communication around the globe. This makes English one of the most spoken and demanded languages worldwide. Thus, learning it opens a wide range of possibilities to its learners, due to the possibility of accessing more information about what is happening in real time in the world.

These places the practitioners in a privileged and advantageous position, for example, to optimize the use of the Internet and its many tools, which often exposes its users to the need for interaction in this language. Likewise, having the ability to establish communication and interact with other people through English facilitates participation, involvement and sensitivity with other cultures, traditions, customs, ideologies, policies and perspectives existing in different countries; but mainly, it improves the possibilities of getting involved in the diversity of educational, academic and work environments, with a global outlook.

In any case, it makes it easier to get to know and integrate into other personal or work groups, allowing for the unwinding to get used to new experiences.

While, on the other hand, the area of mathematics functions as a dynamic and intercommunicating bridge between the different sciences and disciplines of knowledge, serving as a gear for the understanding and optimal functioning of the world, that is, it is from it, that it is possible to articulate science as a whole, producing through this framework, the scientific, technological, social and cultural development of nations (Abancin, 2022a y 2022b). Thus, mathematics as well as English are fundamental pillars for nations to be at the forefront of the world in terms of scientific, technological, social, cultural, among others.

It is in this sense that the usefulness and influence of these two areas at a social level, reveal the importance and need to strengthen their studies from the stage of academic and professional training. Thus, the latter allows focusing the guidelines of the interests of this book, namely, to learn English while studying some fundamental topic of mathematics. This with a view to contributing to the construction of a solid academic formation through the articulation of both disciplines, which have a favorable impact on the professional, improving the possibilities of professionals to incorporate a much broader sphere of vision regarding the world of work.

This is due to the fact that, with the mastery of the English language within the area of mathematics, it provides them with a key tool that will successfully help to break down and expand the boundaries of the work delimited by the Spanish language.

Specifically, the purpose of this book is to crystallize a didactic document that strengthens the teaching and learning process of the English language for mathematicians, supported by the articulation of the English contents with the study of the fundamental topics of mathematical logic, in order to consolidate a suitable level of English in the students of the programs related to mathematics careers and according to the standards of a graduate as a mathematics professional.

Thus, this book is a brief introduction to the fundamentals of mathematical logic, since it is an appropriate subject for a student to begin his or her studies in mathematics. That is, in the first stage of the studies of an undergraduate mathematics program (third level) students need to know the language of mathematical logic. Once they are familiar with these, they can successfully face other aspects of mathematics such as the logical rigor of the reasoning used to justify the results (Uzcátegui, 2011). It is in this sense that, although this book is a tool for the English course, it was written trying to preserve as much formality as possible when presenting the contents referring to the study of the fundamentals of mathematical logic.

Thus, the importance of having English fluency skills in academic and professional training as a mathematician is indisputable, since having a consolidated knowledge of this language as part of the background of this professional is a key point that will open the doors to many opportunities. Among the main ones are, on the one hand, having a more diverse academic background due to the possibility of studying with textbooks and articles in the English language derived from authors and researchers of international level; On the other hand, communication skills in this language will help to establish conversations with people anywhere in the world, thus expanding the network of contacts to build personal, academic and professional relationships, consolidating a circle of intellectual character that will be favored at the time of getting possible opportunities for inter-institutional collaboration that lead to the development of mathematics, and therefore, in the scientific, technological, social, cultural, among others; also, contribute to professional development as a qualified mathematician with an international perspective.

With respect to the justification, it is worth mentioning that currently almost all potential employers for graduates of a mathematics career, such as educational institutions (basic and university), industries and companies, mention as a desirable requirement to look for candidates who speak fluent English. For this reason, for the students of the mathematics career of the Escuela Superior Politécnica de Chimborazo (ESPOCH), the present book framed in an innovative proposal to articulate English with mathematics will give them the opportunity to learn English while studying the fundamentals of mathematical logic. In addition, this book presents some of the ideas and tools involved with mathematical logic that every mathematics student needs to know during the course of their undergraduate studies. This will contribute from their undergraduate studies to educate them in the English language as early as possible, with the possibility of putting it into practice in what will be their area of expertise once they graduate from the mathematics program. Results that will result in a preventive preparation for opportunities that arise both academically and in the work place both nationally and internationally.

For example, with respect to academic opportunities, knowing English and mathematics will provide students with a variety of options for a fourth level academic education (postgraduate) without the need to limit themselves to national Higher Education institutions, but also to look at possibilities abroad, for access to specialized, updated and complete education and information, of course, this will depend on the interests of the trainees. Moreover, English is not only exclusive to study in English-speaking countries, but also in any country in the world, since it is already very common to find educational institutions that offer multiple programs (bachelor's degrees, specializations, master's degrees and doctorates) in English. As well as professional opportunities in terms of work, or simply, to know and have access to the latest advances through scientific, academic and technological texts and articles that are written in English, placing them at the forefront of the mathematical profession.

Finally, the book is structured in sixteen (16) lessons for the study of the English language corresponding to the program of the course of the first Ordinary Academic Period (PAO) of the mathematics career of ESPOCH through the fundamentals of mathematical logic. Specifically, the first lesson introduces the concepts of natural and artificial languages to lay the foundations for a formal study of the fundamentals of logic, with emphasis on knowing and differentiating between metalanguage and object language. The second deals with the conceptions of syntax and semantics as central aspects to be taken into account for the study of mathematics. Lesson 3 is devoted to present some conceptual elements involved in the study of logic, such as assertions, premises, inferences, reasoning and argumentation. Lesson four briefly discusses the meanings of formal and informal logic.

LESSON 1: LANGUAGES INVOLVED IN THE STUDY OF LOGIC.

TASK 1. ANSWER THE QUESTIONS

1. What is the difference between natural language and artificial language?

____

____

____

2. What is the difference between metalanguage and object language?

____

____

____

TASK 2. READ AND CHECK YOUR ANSWERS IN TASK 1.

Language can be classified as informal (natural) and formal (artificial). **Informal language** is commonly used and arbitrary, it can be ambiguous, confusing, vague, with metaphors and decontextualized; while **formal language** is characterized by its precision, being strictly defined in a context in such a way that it does not give rise to doubts or inaccuracies in the veracity of its meaning.

TASK 3. MATCH THE WORD AND THE DEFINITION

a.	Natural and artificial language	…………… The metalanguage is the one used to express oneself when studying another language, that is to say, it is the one used to talk about and describe aspects of a new language.
b.	Metalanguage	…………….The target language is the language that is the focus of study based on another language.
c.	Object language	……………. Natural (everyday or colloquial) languages are those informal languages that are commonly used by human beings for the purpose of oral or written communication.

TASK 4. READ AND WRITE MORE EXAMPLES

NATURAL LANGUAGES EXAMPLES.

- English is the most widely used language worldwide.
- Spanish is a language spoken in most of the countries of South America.
- ____
- ____
- ____

METALANGUAGE EXAMPLES.

- Mother tongue is the metalanguage for students to learn mathematics.
- Spanish is the metalanguage for Spanish-speaking students to learn English.
- Elementary mathematical notation is the metalanguage for the study of the language of set theory.
- ____
- ____
- ____

TASK 5. READ AND WRITE TRUE OR FALSE

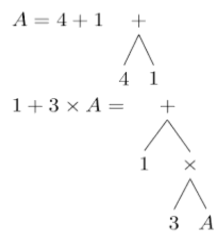
1. The target language is the language that is the focus of study based on another language. _____________
2. The mother tongue is the students' metalanguage for learning mathematics. It is an example of metalanguage _____________
3. The target language is the language that is the focus of study based on another language. _____________

4. If a person whose mother tongue is Spanish is learning the English language, it will be said that the target language is English and that the Spanish language with which expresses from English is the metalanguage. ____________
5. If a Russian student is learning geometry, it will be said that the object language is geometry and the language with which geometry is expressed is metalanguage. This metalanguage can the Russian language (natural) and mathematical notation (artificial____________

TASK 6. READ THE PARAGRAPH AND CIRCLE AND CORRECT THE MISTAKES

Informal language are commonly used and arbitrary, it can to be ambiguous, confusing, vague, with metaphors and decontextualized; while **formal language** are characterized by its precision, being strictly defined in a context in such a way that it do not gives rise to doubts or inaccuracies in the veracity of its meaning.

LESSON 2: SYNTAX AND SEMANTICS IN THE STUDY OF LOGIC



TASK 1. READ SYNTAX AND SEMANTICS.

SYNTAX AND SEMANTICS.

In the area of mathematics there are two central aspects that must be taken into account in order to correctly understand definitions, results such as theorems and procedures carried out within a theory; namely, syntax and semantics.

<u>SYNTAX</u>

$$I = A \cdot \cos \phi$$

Mathematical syntax deals with the way in which mathematics is written, taking into account all formal considerations, but in itself does not allow the attribution of meanings.

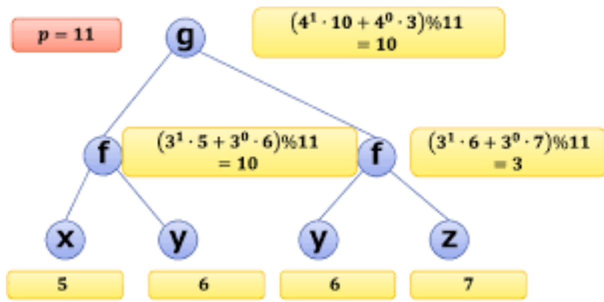
Example 2.1. Syntax of a mathematical expression.

1) $2 \in \mathbb{N}$.

2) $A \cap B$.

SEMANTICS

Mathematical semantics the study of the meaning of mathematical linguistic signs. In other words, it is the exposition of the meaning attributable to syntactically well-formed expressions.



TASK 2. READ SYNTAX AND SEMANTICS AND COMPLETE THE SENTENCES

DEALS	ALLOW	WRITTEN	REPRESENTS	ATTRIBUTABLE
	MEANINGS	LINGUISTIC	CONSIDERATIONS	

- Mathematical syntax____________with the way in which mathematics is____________taking into account all formal____________but in itself does not____________the attribution of____________
- Mathematical semantics______________the study of the meaning of mathematical ______________signs. In other words, it is the exposition of the meaning ______________to syntactically well-formed expressions.

TASK 3. WRITE DOWN SOME MATHEMATICAL EXPRESSIONS.

TASK 4. READ THE EXAMPLES AND WRITE DOWN THE MATHEMATICAL EXPRESSIONS.

Semantics oy Sintax

1) The meaning of the mathematical expression 2∈N is: 2 is a natural number. ________________

2) The meaning of the mathematical expression A∩B is: are all the elements that are in A and in B at the same time ________________

TASK 5. ANALYSIS OF SYNTAX AND SEMANTICS OF A WORD. CONSIDER THE WORD BLOCK. ANALYZE THE SYNTAX AND SEMANTICS OF THIS WORD.

In mathematics, syntax (the way it is written) is very important because a small change can change the semantics (the meaning).

Analyze the syntax and semantics of BLOCK.

TASK 6. DO THE FOLLOWING EXERCISES. DIFFERENT SYNTAXES, BUT THE SAME SEMANTICS.

1) Consider the number: five and 5, written in two different ways. Analyze the syntax and semantics.

2) Consider the multiplication of 2 (two) by 3 (three) in the following ways:
(2)(3) =6, 2×3 =6, 2·3 =6 y 2*3 = 6.
Analyze the syntax and semantics.

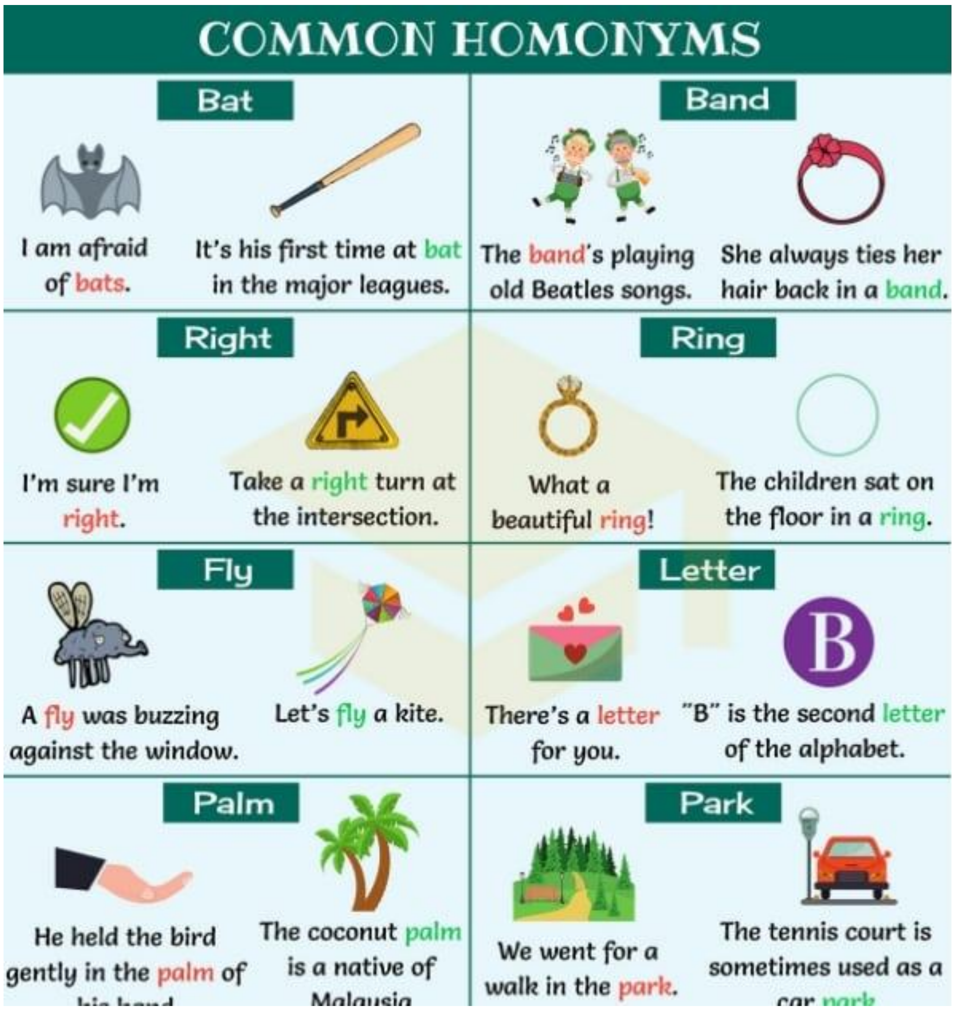
3) Consider the multiplication of 2 (two) by 3 (three) in the following ways:

(2)(3) =6, 2×3 =6, 2·3 =6 y 2*3 = 6.

Analyze the syntax and semantics.

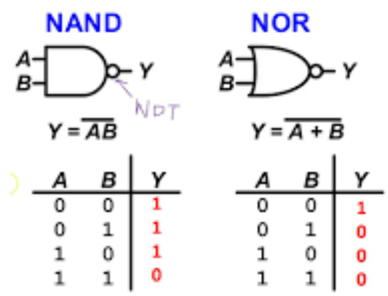
TASK 7. LOOK AT THE CHART AND PRONOUNCE THE WORDS

HOMONYMS two or more words having the same spelling or <u>pronunciation</u> but different meanings and origins.

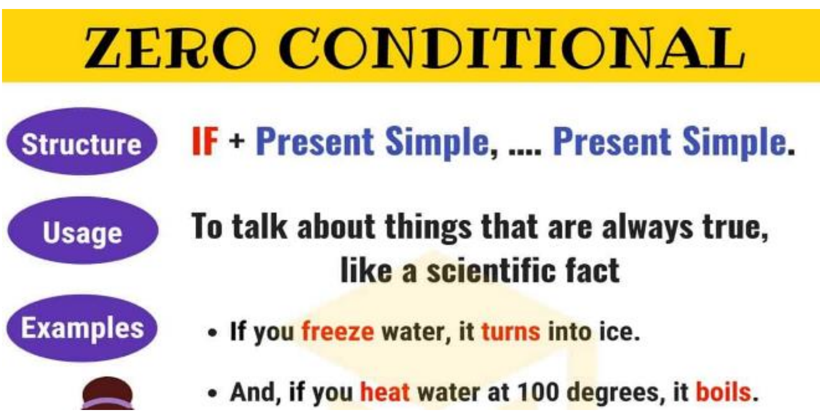


TASK 8. WRITE DOWN HOMONYMOUS WORDS IN ENGLISH AND ANALYZE THEIR SYNTAX AND SEMANTICS.

LESSON 3: SOME CONCEPTUAL ELEMENTS INVOLVED IN THE STUDY OF LOGIC.



GRAMMAR



The zero conditional is used to make statements about the real world, and often refers to general truths, such as scientific facts. In these sentences, the time is now or always and the situation is real and possible.

Conditions

An assertion is the presentation of a statement that is considered valid and is supported by evidence or conviction.

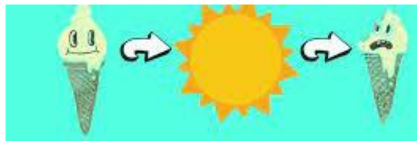
The following are illustrations of assertions.

1) An equilateral triangle has three equal angles.

2) Any real number other than zero raised to zero is equal to one.

3) A single straight line passes through two distinct points.

TASK 1. LOOK AT THE PICTURES AND WRITE ASSERTIONS OR CONDITIONS

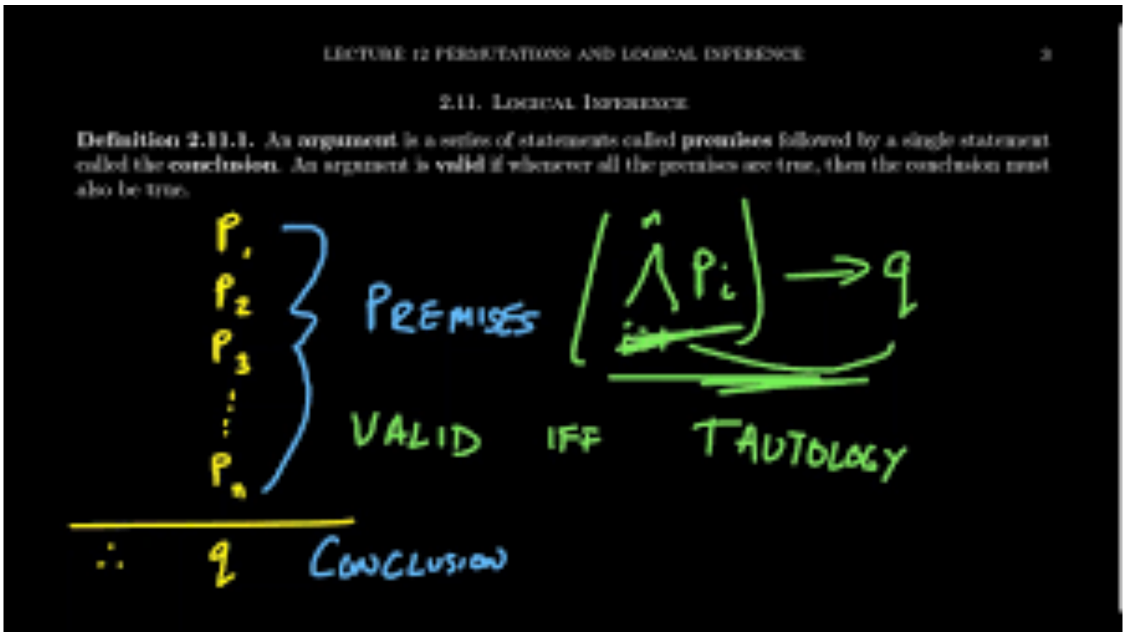


TASK 2. WRITE DOWN MORE EXAMPLES

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____

TASK 3. PREMISES, INFERENCES, REASONING AND ARGUMENTATION.

COMPLETE THE CONCEPTS USING THE WORDS IN THE TABLE.



REASONING	**INFERENCE**	**REASONING**	**ARGUMENTATION**	**ASSUMPTIONS**

1. ________________________are part of a group of statements that provide arguments to derive and defend a conclusion.

2. ________________________is the process by which conclusions are drawn from a set of initial premises.

3. ________________________consists of obtaining statements (conclusions) from other statements (premises) with the appropriate criteria so that one can be sure that if the premises are true, then the conclusions obtained are also true.

4. ________________________is based on a person's ability to process information, connect ideas and use available knowledge to coherently arrive at a valid conclusion, solution or idea. That is, it is a process in which relationships are established from a set of statements to infer another statement.

There are two ways of reasoning: by induction or by deduction. The former consists of obtaining general conclusions from particular premises; whereas, the latter relies on general premises to obtain particular conclusions. Moreover, a reasoning is correct if every time the initial statements are true, then the inferred statement is also true. Thus, the focus of reasoning methods is on not accepting erroneous conclusions (Institute of Mathematical Sciences [ICM], 2006). Therefore, reasoning of the correct type is of interest to the area of mathematics.

1. ________________________consists of an oral or written text whose purpose is to demonstrate, through coherent reasoning, the validity of a point of view, opinion, attitude, behavior or perspective, while persuading the interlocutors.

Reasoning is the ability to express an argumentative sequence to establish new relationships between the units of information that constitute a concept, that is, to derive some concepts from others or to imply a new relationship on the basis of the relationships already established (Rico, 1997); whereas, argumentation is the means through which these relationships are expressed.

TASK 4. READ PREMISES, INFERENCES, REASONING AND ARGUMENTATION AND ANSWER THE QUESTIONS.

1. How many ways of reasoning are there?

2. What is reasoning

3. What is inference?

4. What is reasoning?

5. What is argumentation?

6. What is assumptions?

TASK 5. COMPLETE THE NEXT EXERCISES. ILLUSTRATIONS OF REASONING.

1) If you have premise
Premise 1: "birds sing in the morning" and premise
Premise 2: "the canary is a bird",
What can you conclude?

	PREMISES
Premise 1	
Premise 2	
Conclusion	

2) If we have premise
Premise 1: "the sum of the internal angles of a triangle is 180º" and premise
Premise 2: "an equilateral triangle has all its internal angles equal",
What can we conclude?

	PREMISES
Premise 1	
Premise 2	
Conclusion	

3) If we have premise
Premise 1: Garlic is a vegetable and grows well in autumn.

Premise 2: Lettuce is a vegetable and grows well in autumn.
What can we conclude?

	PREMISES
Premise 1	
Premise 2	
Conclusion	

4) If we have premise
Premise 1: All physicists are very intelligent people.
Premise 2: Albert Einstein was a physicist.

	PREMISES
Premise 1	
Premise 2	
Conclusion	

4) If we have premise

Premise 1: Don Quixote de la Mancha is a classic work and is one of the best books of world literature.

Premise 2: Romeo and Juliet is a classic work.

Conclusion: ____.

	PREMISES
Premise 1	
Premise 2	
Conclusion	

5) If we have premise

Premise 1: People who play sports have a lot of physical endurance.

Premise 2: Jose plays sports.

Conclusion: ____.

	PREMISES
Premise 1	
Premise 2	
Conclusion	

6) If we have premise

Premise 1: Legumes are good for the body.

Premise 2: Chickpeas are legumes.

Conclusion: Chickpeas are good for the body.

	PREMISES
Premise 1	
Premise 2	
Conclusion	

7) If we have premise

Premise 1: Human beings communicate using language.

Premise 2: Peter is a human being.

Conclusion:____.

	PREMISES
Premise 1	
Premise 2	
Conclusion	

GENERAL REVIEW

Although the central idea of both inference and reasoning is to obtain a consequence or derive a conclusion from initial statements, the use of either can make a difference. On the one hand, inference alludes to obtaining conclusions almost immediately and without much intellectual effort; whereas, reasoning refers to a more complex construction that demands intellectual effort to crystallize through the articulation of a set of statements to obtain another statement as a conclusion.

LESSON 4: LOGIC

Logic is the science that deals with the study of reasoning. In other words, logic provides principles and methods that, when applied to the structure of reasoning, allow us to say whether these are correct or not (Muñoz, 1992). In particular, a large part of logic is devoted to the study of correct reasoning (Uzcátegui, 2011).

FORMAL LOGIC *is the logic of demonstration,* ***INFORMAL LOGIC*** *is the logic of argumentation.*

When the demonstration is ***CORRECT OR INCORRECT****, valued in the first case and valueless in the second, the arguments are more or less strong, more or less pertinent, more or less convincing.*

TASK 1. READ FORMAL AND INFORMAL LOGIC AND **FILL IN THE BLANKS WITH THE FOLLOWING VERBS.**

FORMAL AND INFORMAL LOGIC

BE	**INFERRED**	**ALLOWS**	**DEVOTED**	**FOCUSED(2)**	**PROVIDED**
			CONSTRUCTED		

In general terms, logic can ________________- divided into formal (dialectical) and informal (non-formal). The former is ________________- to the arguments present in demonstrations, where the interest is ________________- on whether they are constructed in accordance with the rules of reasoning to determine whether they are correct or incorrect. The second one is ________________- on the study of argumentation within natural languages and everyday thinking in any field. This ________________- solving controversies, justifying an action and making a reasonable decision based on obtaining ________________- conclusions from previously ________________- information.

TASK 2. READ FORMAL AND INFORMAL LOGIC AND **COMPLETE THE CHART**

FORMAL LOGIC	INFORMAL LOGIC

TASK 3. READ LOGIC AND ORDER THE SEQUENCE

Logic is a process for making a conclusion and a tool you can use.

______ Finally, a conclusion is drawn

______The argument is then built on premises.

______The proposition is either accurate (true) or not accurate (false).

______The foundation of a logical argument is its proposition, or statement.

______Premises are the propositions used to build the argument.

______Then an inference is made from the premises.

TASK 4. IN PAIRS TALK ABOUT

“THE DIFFERENCE BETWEEN FORMAL AND INFORMAL LOGIC”

TASK 5. WHAT ARE THE DIFFERENCES BETWEEN FORMAL AND INFORMAL LOGIC?

TASK 6. READ MATHEMATICAL LOGIC AND ANSWER THE QUESTION: WHAT IS LOGICAL MATHEMATICAL AND EXAMPLES?

It is a branch of logic concerned with the study of the fundamental principles of mathematical reasoning and argumentation. It is used to analyze and evaluate the validity of reasoning and to develop formal methods useful for the execution of proofs of mathematical theorems. Logic is also an area of mathematics.

Mathematical logic uses propositional variables, which are often letters, to represent propositions.

WHAT IS LOGICAL MATHEMATICAL?

IS IT USED FOR?

EXAMPLES

TASK 7. READ INFORMAL LOGIC AND WRITE THE EXPLANATION

Informal logic is what's typically used in daily reasoning. This is the reasoning and arguments you make in your personal exchanges with others.

- Premises: Nikki saw a black cat on her way to work. At work, Nikki got fired.

Conclusion: Black cats are bad luck.

Explanation: ____

- Premises: There is no evidence that penicillin is bad for you. I use penicillin without any problems.

Conclusion: Penicillin is safe for everyone.

Explanation: ____

- Premises: My mom is a celebrity. I live with my mom.

Conclusion: I am a celebrity.

Explanation: ____

TASK 8. READ FORMAL LOGIC AND FOLLOW THE PREMISES TO REACH A FORMAL CONCLUSION.

FORMAL LOGIC you use deductive reasoning and the premises must be true. You follow the premises to reach a formal conclusion.

- Premises: Every person who lives in Quebec lives in Canada. Everyone in Canada lives in North America.

Conclusion: Every person who lives in Quebec lives in North America.

Explanation: ____

- Premises: All spiders have eight legs. Black Widows are a type of spider.

Conclusion: Black Widows have eight legs.

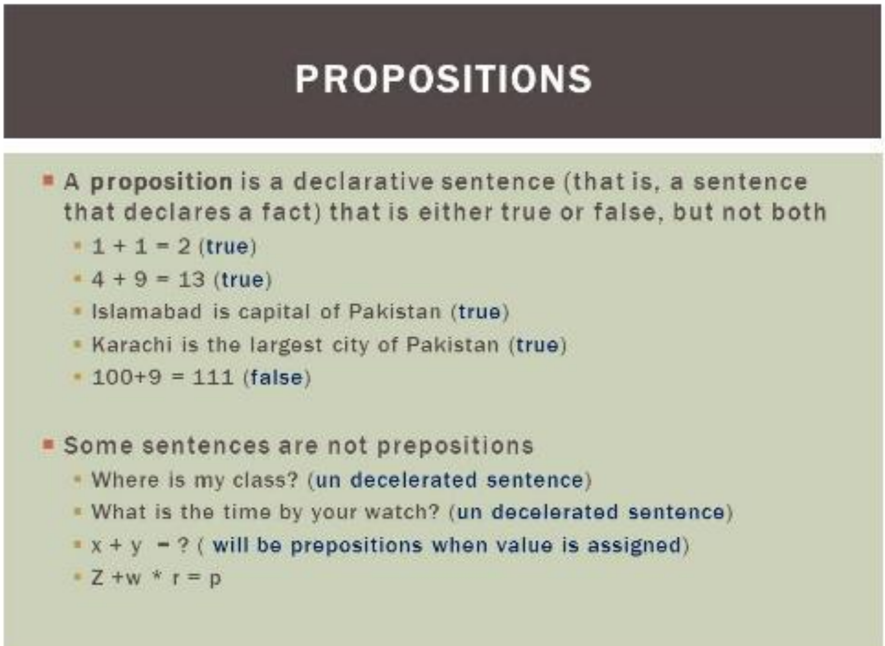
Explanation: ____

- Premises: Bicycles have two wheels. Jan is riding a bicycle.

Conclusion: Jan is riding on two wheels.

Explanation: ____

LESSON 5: PROPOSITIONS



This kind of sentences are called propositions. If a proposition is true, then we say it has a truth value of "true"; if a proposition is false, its truth value is "false", but not both at the same time.

The assertion characteristic of propositions marks the fundamental difference between other types of sentences such as questions, commands, exclamations, since only propositions can be judged as true or false.

For example:

"Grass is green",

and "2 + 5 = 5" are propositions.

The first proposition has the truth value of "true" and the second "false".

TASK 1. ARE THE NEXT SENTENCES PREPOSITONS? WRITE TRUE OR FALSE.

1. "7 is a prime number". ______________________________
2. "46-17=29". ______________________________
3. " Quito is the capital of Ecuador______________________________
4. "all integers are negative". ______________________________
5. "dogs fly______________________________

6. "$x^2+5x+6=0$, for all $x \in R$". ______________________________
7. "Manuel is in college". ______________________________
8. "the square of every even number is also even". ______________________________
9. "Ramon bought a house______________________________

TO CONSIDER

It is noted that all of the above sentences can be qualified accurately and without ambiguity or subjectivism, since they are either true or false.

TASK 2. WRITE DOWN MORE EXAMPLES

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____

TASK 3. READ UNDECIDABLE PROPOSITIONS AND WRITE DOWN MORE EXAMPLES

The fundamental elements of logic are propositions. Therefore, sentences that are neither false nor true, sentences that are false and true at the same time, or sentences that show some kind of imprecision (lack of meaning), are not the object of study of logic (ICM, 2006, p. 9).

The following sentences are not propositions.

1) Let's play chess.

2) Take care of yourself!

3) What day is it today?

4) How are you?

5) Hurry up!

6) $a+b=a$.

7) The square of a whole number is even.

8) Why do you study mathematics?

9) He is a student.

TASK 4. WRITE MORE EXAMPLES

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________

GENERALLY, IMPERATIVE, EXCLAMATORY AND INTERROGATIVE SENTENCES ARE NOT PROPOSITIONS.

TASK 5. READ THE SENTENCES AND COMPLETE THE CHART BELOW. WHICH ARE PROPOSITIONS AND WHICH AREN'T

Two plus two is four. Two plus two is zero. Bicycles have three wheels. "How are you doing?"

"Picasso's painting Guernica is obedient." "Picasso's painting Guernica is beautiful." "He is an Olympic swimmer. "Michael Phelps is an Olympic swimmer."

PROPOSITIONS	NO PROPOSITIONS

SYMBOLIC REPRESENTATION FOR PROPOSITIONS

TASK 6. BEFORE READING: HOW CAN BE PROPOSITIONS REPRESENTED ?

In general, the declarative sentence with which a proposition is expressed can be long and complex, and for this reason it is convenient, in order to simplify its presentation and manipulation, to replace it by a letter.

In this sense, propositions can be represented by lowercase letters of the alphabet, i.e., a,b,c,⋯,p,q,r,⋯,x,y,z.

Symbolic representation of the propositions

1) a: "7 is a prime number".

2) b: " 46-17=29".

3) c: "Quito is the capital of Ecuador".

4) d: "all integers are negative".

5) e: "dogs fly".

6) f: "$x^2+5x+6=0$, for all $x \in R$".

7) g: "I am in college".

8) h: "the square of every even number is also even".

TASK 7. WRITE DOWN MORE EXAMPLES

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________
8. ______________________________
9. ______________________________
10. ______________________________

PROPOSITIONAL LOGIC

	Determine the type of Sentence	If a proposition determine its truth value
5 is a prime number.	Declarative and Proposition	T
8 is an odd number.	Declarative and proposition	F
Did you lock the door?	Interrogative	
Happy Birthday!	Exclamatory	
Jane Austen is the author of Pride and Prejudice.	Declarative and Proposition	T
Please pass the salt.	Imperative	
She walks to school.	Declarative	
$\vert x+y\vert \leq \vert x\vert + \vert y\vert$	Declarative	

Formal properties of propositions and rules of inference, all this in order to simplify the learning of methods for making demonstrations that are an essential tool for the study of mathematics (Uzcátegui, 2011).

TASK 8. IDENTIFY WHICH OF THE STATEMENTS ARE PROPOSITIONS

1) No smoking.

2) x2+y2≥9

3) Court in Lima will hear complaints about Ancash.

4) 4x - 1= - 5

5) What time is it?

6) Ruling against mega-commission confronts the Judiciary and the Congress

7) He is a student of the Faculty of Administrative and Accounting Sciences.

8) - 6.78 > 1.43

9) Unemployment fell slightly in February

10) Help!

11) Stop.

12) Ollanta Humala is not the president of Peru.

13) Paolo guerrero is a soccer player.

14) Where were you?

15) No noise allowed

16) Judge annuls all the reports accusing Garcia.

Investigate!

You stumble upon two trolls playing Stratego. They tell you:
Troll 1: If we are cousins, then we are both knaves.

Troll 2: We are cousins or we are both knaves.

Could both trolls be knights? Recall that all trolls are either always-truth-telling knights or always-lying knaves.

TASK 9. TRANSLATE THE FOLLOWING ENGLISH SENTENCES TO PROPOSITIONAL LOGIC.

Propositions: (R)aining, Liron is (S)ick, Liron is (H)ungry, Liron is (HA)appy,

Liron owns a (C)at, Liron owns a (D)og

(a) It is raining if and only if Liron is sick

(b) If Liron is sick then it is raining, and vice versa

(c) It is raining is equivalent to Liron is sick

(d) (d) Liron is hungry but happy

(e) Liron either owns a cat or a dog

TASK 10. zRANSLATE THE FOLLOWING PROPOSITIONAL LOGIC TO ENGLISH SENTENCES.

Let:

- E=Liron is eating
- H=Liron is hungry

(a) $E \Rightarrow \neg H$

Answer: ____

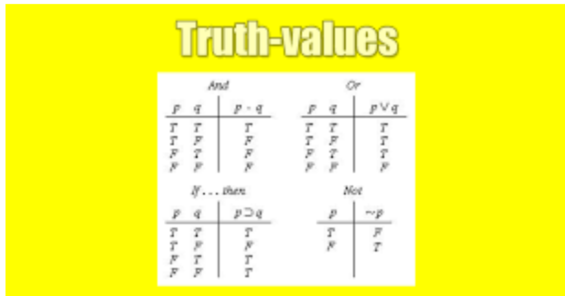
(b) $E \wedge \neg H$

Answer: ____

(c) $\neg(H \Rightarrow \neg E)$

Answer: ____

TRUTH VALUE OF A PROPOSITION



Truth value is the attribute of a proposition as to whether the proposition is true or false.

TASK 11. READ THE SENTENCES AND COMPLETE : TRUE or FALSE

- The truth value for "7 is odd" is ________________ which can be denoted as T.
- The truth value of "1 + 1 = 3" is ________________ which can be denoted as F.

In order to reason correctly, it must be guaranteed that from true propositions another true proposition is inferred.

For this reason, it is essential to be able to decide when a proposition is true.

TASK 12. MATCH THE WORD AND THE CORRECT DESCRIPTION

When the proposition is true,	**a. it is said to have a false truth value (false proposition) and is associated with: 0 or F.**
when it is false,	b. it is said to have a true truth value (true proposition) and is associated with: 1, V or T

Any of the above notations could be used, but the convention to be followed in this book will be the use of 0 and 1.

The truth and falsity values of propositions are considered as the logical values and are the most interesting values of a proposition.

TASK 13. READ TRUTH VALUE OF A PROPOSITION AND ANSWER THE QUESTIONS.

1. What is the truth value of a preposition?

2. Why is it is essential to be able to decide when a proposition is true.

3. Is the truth value for "7 is odd" true, which can be denoted as T. Why? Explain it.

4. How is the true preposition represented?

5. How is the false preposition represented?

TASK 15. READ THE EXAMPLES AND WRITE TRUE OR FALSE IN EACH PROPOSITION.

- The truth value for proposition a: "7 is a prime number", is ____________________
- The truth value for proposition b: "46-17=29", is ____________________
- The truth value for proposition c: "the city of Quito is the capital of Ecuador", is ____________________
- The truth value for proposition d: "all integers are negative", is ____________________
- The truth value for proposition e: "dogs fly", is ____________________
- The truth value for proposition f: "x^2+5x+6=0, for all x∈R", is ____________________
- The truth value for proposition h: "the square of every even number is also even", is ____________________

TASK 16. FIND THE TRUTH VALUE OF ∀? (?3 > 0)FOR EACH OF THE FOLLOWING DOMAINS. IF THE STATEMENT IS FALSE, PROVIDE A COUNTEREXAMPLE.

(a)? ∈ **Z** +

(b)? ∈ **R**

(c)? ∈ **R** +

(d)? ∈ **Z**

TASK 17. Determine whether each of the following sentences is a proposition. If it is a proposition, determine whether it is true or false.

- "If and only if I win the lottery, then I will buy a house." ___________________
- All cows are brown. ___________________
- The Earth is further from the sun than Venus ___________________
- There is life on Mars ___________________
- "If the weather is good, people go to the beach" ___________________
- George Washington was the first U.S. president. ___________________
- Scooby Doo, where are you? ___________________
- Fort Worth is the capital of Texas. ___________________
- Look both ways before crossing the road. ___________________
- Tobey Maguire was the best actor to play as spiderman. ___________________
- There are an even number of stars in the universe. ___________________
- A is a vowel and B is a consonant. ___________________
- A is a vowel or B is a consonant. ___________________
- Pigs can fly, but dolphins can swim. ___________________
- Pigs can fly or dolphins can swim. ___________________
- 371 is an even number and 426 is an odd number. ___________________

TRUTH TABLE OF A LOGICAL EXPRESSION

TASK 18. READ AND COMPLET WITH THE GIVEN WORDS

TRUTH TABLE **LOGICAL EXPRESSION** **NUMBER OF COMBINATIONS**

- A________________________________ can be any expression that evaluates to true or false, i.e. a logical expression returns 1 if the expression is true, and 0 if it is false.

- A________________________________ is a representation of the possible truth values that a proposition could take. Thus, truth tables serve to show the values, relationships and possible results when performing operations with logical expressions.

-
- The ________________________________ (rows of the truth table) depends on the number of propositions present in the logical expression. Therefore, we have the following formula

Number of rows=2^n, where n is the number of propositions.

Truth table for a logical expression with n propositions a_1,a_2,⋯,a_n

Propositions

2^n

combination

a_1	a_2	$\cdots$	a_n	Logical expression
0	**0**	$\cdots$	**0**	
0	**1**	$\cdots$	**1**	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
1	**1**	$\cdots$	**0**	
1	**1**	$\cdots$	**1**	

TASK 19. CONSTRUCT A TRUTH TABLE FOR LOGICAL EXPRESSIONS FOR:

1. Truth table for a logical expression where only one proposition a is present.

Truth table for a logical expression with only one proposition a.

a	Logical expressions
0	
1	

2. Truth table for a logical expression where two propositions a and b are present.

Truth table for a logical expression with two propositions a and b.

a	b	Logical expressions
0	**0**	
0	**1**	
1	**0**	
1	**1**	

3. Truth table for a logical expression where three propositions a, b and c are present.

Truth table for a logical expression with three propositions a, b and c.

a	b	c	Logical expressions
0	**0**	**0**	
0	**0**	**1**	
0	**1**	**0**	
0	**1**	**1**	
1	**0**	**0**	
1	**0**	**1**	
1	**1**	**0**	
1	**1**	**1**	

4. Truth table for a logical expression where only one proposition a is present.

Truth table for a logical expression with only one proposition a.

a	Logical expressions
0	
1	

5. Truth table for a logical expression where two propositions a and b are present.

Truth table for a logical expression with two propositions a and b.

a	b	Logical expressions
0	**0**	
0	**1**	
1	**0**	
1	**1**	

6. Truth table for a logical expression where three propositions a, b and c are present.

Truth table for a logical expression with three propositions a, b and c.

a	b	c	Logical expressions
0	**0**	**0**	
0	**0**	**1**	
0	**1**	**0**	
0	**1**	**1**	
1	**0**	**0**	
1	**0**	**1**	
1	**1**	**0**	
1	**1**	**1**	

TASK 20. CONSTRUCT A TRUTH TABLE FOR EACH OF THE FOLLOWING COMPOUND PROPOSITIONS.

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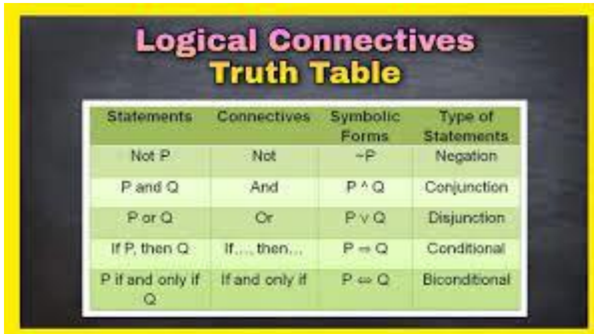
LESSON 6: LOGICAL CONNECTIVES


Logical Connectives
Truth Table

Statements	Connectives	Symbolic Forms	Type of Statements
Not P	Not	~P	Negation
P and Q	And	P ^ Q	Conjunction
P or Q	Or	P ∨ Q	Disjunction
If P, then Q	If…, then…	P ⇒ Q	Conditional
P if and only if Q	If and only if	P ⇔ Q	Biconditional

Logical or propositional connectors (or logical operators or links) are words, expressions or symbols that make it possible to link any propositions to obtain new propositions.

The five words or phrases that will be used within the fundamentals of logic as connectors are: "not", "and", "or", "if ···, then···" and "if, and only if". Each of these is associated with a logical connector: negation, conjunction, disjunction, conditional and biconditional, respectively. The following table shows how these connectives will be represented in symbolic form.

TASK 1. COMPLETE THE TABLE WITH THE CORRECT NAME

MEANING	SYMBOL	CONNECTIVE
NOT	¬	Negation
AND	∧	Conjuntion
OR	∨	Disjunction
IF/THEN	→	Condiitonal
IF AND ONLY IF	↔	Biconditional

TASK 2. MATCH THE WORD AND THE DEFINITION

a. Negation	______________of P and Q, denoted P∧Q,∧, is the proposition "P and Q.." P∧Q ∧ is true exactly when *both* P and Q are true.
b. Conjunction	______________of P and Q, denoted PvQ,v, is the proposition "P or Q.." PvQv is true exactly when *at least one* P or Q is true.
c. Disjunction	______________of P, denoted ¬P,¬, is the proposition "not P.." ¬P¬ is true exactly when P is false.

Negation:

Let p be a proposition. The negation of p is the statement "it is not the case that p."

- The negation of p is denoted¬?.
- ¬?has the opposite truth value of p.
- Propositions can be negated by inserting or removing a negation word or phrase, such as "not," "no," "none," "neither... nor," etc.

TASK 3 IDENTIFY SIMPLE PROPOSITIONS AND THE LOGICAL CONNECTORS PRESENT.

- "I didn't find you at your job".

The simple proposition is: ________________________________ and the logical connector is: ________________________________ (¬).

- "Alejandro went to the University and it was closed".

The simple propositions are: ________________________________ and ________________________________ and the logical connector is: ________________________________ (∧).

- "Ramon's car is either purple or gray".

The simple propositions are: ______________________________ and ______________________________ and the logical connector is: "o______________________________ (v).

- "If I win the lottery, then I buy a house".

The simple propositions are: ______________________________ and ______________________________ and the logical connector is: ______________________________ (→).

- "I obtain a degree at ESPOCH if, and only if, I make an effort".

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________ (↔).

- The boss gives the orders] and [the employees obey.

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________.

- The referee arrived on time], but [the players did not show up at the stadium.

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________.

- They are going to close], [you must hurry.

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________.

- The writer uses very nice metaphors] and [makes unique descriptions.

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________.

- The waiter took the orders and the food arrived in no time.

The simple propositions are: ______________________________ and ______________________________; and the logical connector is: ______________________________.

- If the driver doesn't hurry, we will miss the plane.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

- He suddenly felt very tired] and [a car picked him up.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

- The children don't want to go out], even if [the day is sunny.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

- Martin is coming tomorrow, but his girlfriend doesn't know.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

- My cousin, who I told you about all night, is about to arrive.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

- Better not go out, it's raining heavily and they announced snowfall in the early morning.

The simple propositions are: ______________________ and ______________________; and the logical connector is: ______________________.

TASK 4. CHOOSE THE BEST OPTION

1. No one expected ______ Felix ______ Ben to be selected. They must be extremely happy.

   (A) neither ... nor

(B) both ... and

(C) both ... or

(D) not only ... and

2. I don't think we can make it ______ you pitch in.

(A) If

(B) Unless

(C) And

(D) and yet

3. ______ night fell, the children packed their things and were ready to leave.

(A) Despite

(B) Unless

(C) As

(D) For

4. They did not do well in the physical test. ______, they were accepted.

   (A) Yet
   (B) Consequently
   (C) Thus
   (D) Therefore

5. We mentioned it to him ______ we ran into him outside the school.

   (A) When
   (B) Whenever
   (C) Where
   (D) Wherever

6. They were told to practice more ______ they would face elimination in the final game.

   (A) Or
   (B) And
   (C) So
   (D) But

7. _______ the blue dress ______ the yellow one look cheap. I like neither.

   (A) Either ... or

   (B) Neither ... nor

   (C) Both ... or

   (D) Both ... and

8. ______ we comforted her, she could not seem to get over the bad experience.

   (A) Although

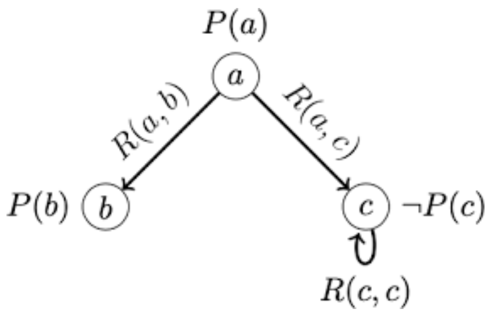
   (B) For

   (C) Since

   (D) Therefore

Within a natural language, more complex propositions are frequently used, not so simple or elementary.

ARITY OF LOGIC CONNECTORS



The link ¬ is a unary connector, that is, the operator ¬ negates only one proposition; whereas, the rest of the connectors (∧,∨, → and ↔) are binary, since they require two propositions to operate.

TASK 5. IDENTIFY THE SIMPLE PROPOSITIONS AND LOGICAL CONNECTORS PRESENT, AS WELL AS, VERIFY THE ARITY OF THE SAME.

- "It is not getting cold".

The simple proposition is: ".................."; and the logical connector is: "............."(   ), verifying that its arity is ...., since, ...

- b) "Alejandra has an apartment and a car."

The simple proposition is: ".................."; and the logical connector is: "............."(   ), verifying that its arity is ...., since, ...

- c) "Manuel studies a mathematics major or a physics major."

The simple proposition is: ".................."; and the logical connector is: "............."(   ), verifying that its arity is ...., since, ...

- d) "If Maria studies education, then she teaches."

The simple proposition is: ".................."; and the logical connector is: "............."(   ), verifying that its arity is ...., since, ...

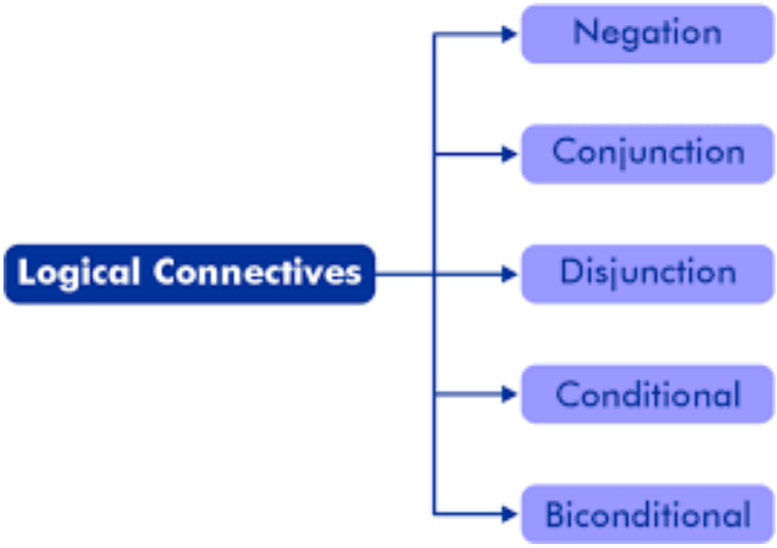
e) "Today is Sunday if, and only if, yesterday was Saturday."

The simple proposition is: ".................."; and the logical connector is: "............."(   ), verifying that its arity is ...., since, ...

TASK 6. FILL IN THE BLANKS WITH SUITABLE LOGICAL CONNECTORS

- Susie refused to take part in the concert. ______, she changed her mind the next day.
- The car beat the red traffic light. ______, the driver was issued a summons by the traffic policeman.
- "Fira won gold medal after three months of intensive training. ______, you too could win if you practice hard enough," Liza's mother said to her.
- "I don't think she can handle this task. ______, she already has a lot of other responsibilities," said the head prefect to his assistant.
- The cadets were given new uniforms to wear. ______, they received free passes to the match.
- The people strongly opposed the move to build a golf course near their house. ______, the proposed plan was cancelled.
- "The final examinations are coming soon. ______, it would be advisable for you to begin revising more systematically," the teacher told her class.
- The neighbors often helped each other, ______ creating a feeling of harmony in the neighborhood.
- The teenager was caught shoplifting. ______, he was let off with a warning.
- Shaun is directing the movie. ______, he is playing the lead role in it.
- Thomas Edison failed several times before he successfully invented the first light bulb. ______, you too could attain your dreams if you never give up trying.
- The bungalow had seven rooms, each with an attached bathroom. ______, there was a large garden and a swimming pool.

LESSON 7: TYPES OF CONNECTIVES: NEGATION



NEGATION

The negation of proposition a, represented symbolically by ¬a, is a new proposition that has truth value true if a is false and has truth value false when a is true.

The connector ¬ is read "not" or "negation". Therefore, ¬a is read "not a" or "negation of a".

The truth table for negation of a proposition a is given by:

Truth table for negation of a proposition a.

a	$\neg a$
0	**1**
1	**0**

1) This logical connector changes the truth value of a proposition:

(a) If a is a true proposition, then ¬a is false;

b) If a is a false proposition, then ¬a is true.

2) Negation is presented with the grammatical terms: "not", "neither", "it is not true that", "it is not true that".

3) Table 3.2 shows the two possible combinations of truth value assignments for proposition a.

4) In natural language, there may be some expressions to indicate the negation of a proposition, such as: "not a", "it is not true that a", "it is not the case that a" and "it is false that a".

The following are negations of propositions.

1) If we have the proposition a: "I have a purple car", the negation of a is ¬a: "I do not have a purple car".

2) If we have the proposition b: "I don't want to study", the negation of b is ¬b: "I want to study".

3) If we have proposition c: "it is raining", the negation of b is ¬b: "it is not raining".

TASK 1. EXPRESS THE NEGATION OF EACH OF THE FOLLOWING PROPOSITIONS. MULTIPLE FORMS OF THE FINAL ANSWER MAY BE POSSIBLE.

(a)Spock is a character in Star Wars.

(b)Earth is not the fourth planet from the sun.

(c)32 = 9.

(d)$2 < 5$.

(e) $-1 \leq 4$

TASK 2. WHAT IS THE NEGATION OF EACH OF THE FOLLOWING PROPOSITIONS?

- Norfolk is the capital of Virginia.
- Food is not expensive in the United States.
- 3 + 5 = 7.
- The summer in Illinois is hot and sunny.
- There is an honest politician.
- All Americans eat cheeseburgers.
- No ferrets drink coffee.
- Not all heroes wear capes.
- Some real numbers are not irrational

TASK 3. COMPLETE THE SENTENCES. USE: EITHER, OR, NEITHER, NOR

1. _______________ the UK _______ Spain are in Asia.
2. Let's meet on _______________ Monday or Tuesday.
3. They weren't at _______________ of the stores.
4. Neither Maria _______ Eduardo was at home.
5. _______________ of the answers is correct. Try again.
6. Either it will rain tomorrow, _______ it won't rain.
7. The hat was _______________ too large, _______ too small. I was the right size.
8. _______________ of the movies were interesting. They were both boring.
9. _______________ of my classmates could come to the party. They were both sick.
10. I don't like _______________ of those two coffee shops.

11. _______________ cats _______ dogs are allowed in the restaurant.

12. We can take a flight at _______________ one o'clock _______ three-thirty.

13. _______________ Vancouver nor Toronto is the capital city of Canada.

14. Either tomorrow _______ the day after tomorrow is a good day to meet.

15. I can't find _______________ of my pencils.

TASK 4. WRITE THE CORRECT PAIR (EITHER / OR AND NEITHER / NOR).

- In this game, you __ win __ lose. It depends on you.
- ___ Sue ___ Sara will help you with your homework. They are both busy at the moment.
- This is my offer. You ___ take it ___ leave it.
- When I go to the restaurant, I eat ___ fish ___ roast chicken. These are my favorite meals.
- His father believed ___ his son ___ his friend. He thought that both were lying.
- I need ___ your help ___ your compassion. I can perfectly handle my problems all alone.
- ___ Charly ___ Bill will write the report. Just ask one of them.
- ___ you return the money you had stolen ____ I'll call the police.
- My mum can ___ read ___ write. She is illiterate.
- You can use ___ this computer ___ the other one. Someone must fix them first.

LESSON 8: TYPES OF CONNECTIVES: CONJUNCTION

CONJUNCTION TRUTH TABLE

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

TASK 1. WHAT IS CONJUNCTION? WRITE A PERSONAL DEFINITION

TASK 2. READ AND CHECK THE DEFINITION

The conjunction between propositions a and b, represented symbolically by $a \wedge b$, is a new proposition that has truth value true if both a and b have truth value true. In other cases, it possesses false truth value.

The connector $\wedge$ is read "and". Therefore, $a \wedge b$ is read "a and b".

The truth table for the conjunction between propositions a and b is given by:

Truth table for the conjunction of two propositions a and b.

a	b	$a \wedge b$
0	**0**	**0**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

1) The proposition $a \wedge b$ is true only when both a and b are true, otherwise $a \wedge b$ is false.

2) The conjunction is presented with the grammatical terms: "and", "but", "more": and punctuation marks such as: comma, period, and semicolon.

3) Table 3.3 shows the four possible combinations of truth value assignments for propositions a and b.

4) In natural language, there may be some expressions to indicate a conjunction between two propositions a and b, such as: "a and b", "a but b", "a nevertheless b" and "a nevertheless b".

5) The word "and" does not always denote a conjunction. For example, the word "and" in the sentence "Manuel and Maria are siblings" does not denote a conjunction, since it cannot be broken down into two propositions.

The following are conjunctions of propositions.

1) If you have the propositions, a: "Alejandra has a house" and b: "Alejandra has a car". The conjunction between a and b is:

a∧b: "Alejandra has a house and a car".

2) If you have the propositions, a: "I studied hard for the math test" and b: "I got a low grade". The conjunction between a and b is:

a∧b: "I studied hard for the math exam but got a low grade".

TASK 4. DO THE EXERCISES

Given:	a: A square is a quadrilateral.
	b: Harrison Ford is an American actor.
Problem:	Construct a truth table for the conjunction "a and b."

SOLUTION:

Given:	r: The number x is odd.
	s: The number x is prime.
Problem:	Can we list all truth values for r^s in a truth table?Why or why not?

SOLUTION:

Given:	p: The number 11 is prime.	true
	q: The number 17 is composite.	false
	r: The number 23 is prime.	true
Problem:	For each conjunction below, write a sentence and indicate if it is true or false.	

SOLUTION:

Construct a truth table for each conjunction below:

1.	x and y
2.	~x and y
3.	~y and x

SOLUTION:

TASK 5 UNDERLINE THE CORRECT ANSWER

1. Which of the following sentences is a conjunction?

Jill eats pizza or Sam eats pretzels.

Jill eats pizza but not pretzels.

Jill eats pizza and Sam eats pretzels.

None of the above.

2. Which of the following statements is a conjunction?

p + q

pq

~p

None of the above.

3. A conjunction is used with which connector?

Not

Or

And

None of the above.

TASK 6. CHOOSE THE CORRECT ANSWER

- I looked, ____ I didn't see him.

and

but

or

as

- She speaks slowly _____ clearly.

and

but

or

both

- She speaks fast, _____ I understand her.

and

but

or

as

- What would you like, coffee _____ tea?

and

but

or

both

LESSON 9: TYPES OF CONNECTIVES. DISJUNCTION

Disjunction:		
p	*q*	*p* v *q*
T	T	T
T	F	T
F	T	T
F	F	F

EXAMPLE: Put the following simple assertions together to make a disjunction:

p: The sun shines.

q: It rains.

ANSWER: The disjunction of the statements p and q is given by

p v q: The sun shines or it rains.

TASK 1. COMPLETE THE CHART

The disjunction of two propositions can be inclusive and exclusive.

INCLUSIVE DISJUNCTION

p	*q*	*p* v *q*
T	T	T
T	F	T
F	T	T
F	F	F

The inclusive disjunction between a and b, represented symbolically by avb, is a new proposition that possesses false truth value if both a and b possess false truth value simultaneously.

The connector v is read "or". Therefore, avb is read "a or b".

The truth table for the inclusive disjunction between propositions a and b is given by:

Truth table for the inclusive disjunction of two propositions a and b.

a	b	$a \vee b$
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**1**

1) The logical connector for **inclusive disjunction** relates two propositions to form a new one, in which the resulting proposition avb will be false only when both a and b possess false truth value, in any other case avb is true.

2) Disjunction is presented with the grammatical terms: "or".

3) In natural language, there may be some expressions to indicate a disjunction between two propositions a and b, such as: 'either a or b' and 'at least a or b'.

In other words, inclusive disjunction is when there is a possibility for both propositions to occur.

TASK 2 TRANSLATE THE FOLLOWING ENGLISH SENTENCES INTO OUR FORMAL LANGUAGE USING CONJUNCTION (THE DOT), NEGATION (THE TILDE), OR DISJUNCTION (THE WEDGE). USE THE SUGGESTED CONSTANTS TO STAND FOR THE ATOMIC PROPOSITIONS.

- Either Bob will mop or Tom will mop.

- It is not sunny today.

- It is not the case that Bob is a burglar.
- Harry is arriving either tonight or tomorrow night.
- Gareth does not like his name.
- Either it will not rain on Monday or it will not rain on Tuesday.
- Tom does not like cheesecake.
- Bob would like to have both a large cat and a small dog as a pet.
- Bob Saget is not actually very funny.
- Albert Einstein did not believe in God.

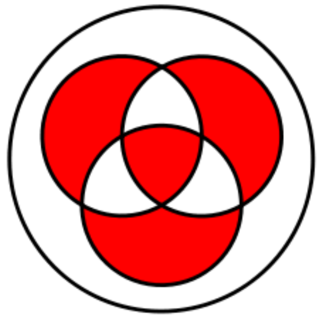
TASK 3. COMPLETE THE FOLLOWING ARE INCLUSIVE DISJUNCTIONS OF PROPOSITIONS.

- If you have the propositions, a: "I have a book on mathematical logic" and b: "I have a book on number theory". The inclusive disjunction between a and b is:
- If one has the propositions, a: "it is raining" and b: "it is getting cold". The inclusive disjunction between a and b is

TASK 4. COMPLETE THE SENTENCES USING OR EITHER

- It was very late, _____________ we took a taxi.
- Would you like to do it now _____________ later.
- I don´t like coffee _____________ Tea.
- You can have _____________ tea _____________ coffee for breakfast. What would you like?
- Sam is _____________ a fool _____________ a stupid.
- She is _____________ Intelligent_____________ hard working.

EXCLUSIVE DISJUNCTION



Let a and b be two propositions. The exclusive disjunction between a and b, symbolically represented by a v b, is a new proposition that possesses false truth value if both a and b possess the same truth value simultaneously.

The connector v is read "or". Therefore, a v b is read "a or b".

The truth table for the exclusive disjunction between propositions a and b is given by:

Table 3.5: Truth table for the exclusive disjunction of two propositions a and b.

a	b	$a \veebar b$
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

1) This logical connector for exclusive disjunction relates two propositions to form a new one, in which the resulting proposition a v b will be false only when both a and b possess the same truth value, otherwise a v b is true.

2) Exclusive disjunction can be presented with the following grammatical terms: "or", "or only", "or only", "or..., or...".

3) In natural language, there may be some expressions to indicate the conditional a→b between propositions a and b, such as "a or b", "a or only b", "a or only b" and "either a, or b".

In other words, exclusive inclusion is when propositions are mutually exclusive of each other.

THE FOLLOWING ARE EXCLUSIVE DISJUNCTIONS OF PROPOSITIONS.

1) If we have the propositions, a: "I am in Ambato" and b: "I am in Riobamba". The exclusive conjunction between a and b is:

a v b: "either I am in Ambato or in Riobamba".

2) If we have the propositions, c: "Carlos was born in January" and d: "Carlos was born in February". The exclusive conjunction between c and d is:

c v d: "Carlos was born in January or February".

It is observed that in the above examples, only one of the propositions involved in the exclusive disjunction can occur.

TASK 5. ANALIZE THE SENTENCES

- They gave money to the Conservative Party either personally or through their companies.

The exclusive conjunction between and is:

- Sightseeing is best done either by tour bus or by bicycle.

The exclusive conjunction between and is:

- Either she goes or I go.

The exclusive conjunction between and is:

- His birthplace was either Newark, New Jersey, or Ohio.

The exclusive conjunction between and is:

- They found no sign of either him or his son.

The exclusive conjunction between and is:

- There were glasses of champagne and cigars, but not many of either were consumed.

The exclusive conjunction between and is:

- If either were killed, delicate negotiations would be seriously disrupted.

The exclusive conjunction between and is:

- They are able to talk openly to one another whenever either of them feels hurt.

The exclusive conjunction between and is:

- Have either of you rented before?

The exclusive conjunction between and is:

- I don't particularly agree with either group.

The exclusive conjunction between and is:

- She warned me that I'd never marry or have children. – I don't want either.

The exclusive conjunction between and is:

- There are no simple answers to either of those questions.

The exclusive conjunction between and is:

- He sometimes couldn't remember either man's name.

The exclusive conjunction between and is:

- He did not even say anything to her, and she did not speak to him either.

The exclusive conjunction between and is:

- 'I'm afraid I've never been there.'—'Well, of course, I haven't myself either.'

The exclusive conjunction between and is:

- Don't agree, but don't argue either.

The exclusive conjunction between and is:

- I can't manage that by myself and I don't see why it should be expected of me either.

The exclusive conjunction between and is:

- The basketball nets hung down from the ceiling at either end of the gymnasium.

The exclusive conjunction between and is:

- I suddenly realized that I didn't have a single intelligent thing to say about either team.

The exclusive conjunction between and is:

- John isn't a liar, but he isn't exactly honest either

The exclusive conjunction between and is:

LESSON 10: CONDITIONALS

Converse of a conditional statement			
Statement	Example	Symbolic form	Read as
Conditional	If a figure is a square, then it has four sides.	$p \to q$	If p, then q
Converse	If a figure has four sides, then it is a square.	$q \to p$	If q, then p

The **conditional** (or implication) between a and b, represented symbolically by a→b, is a new proposition that has a false truth value if a has a true truth value and b a false truth value; whereas, in all other cases the proposition will always be true.

The connector → is read "If...then...". Therefore, a→b is read "If a then b".

The truth table for the conditional between propositions a and b is given by:

Truth table for the conditional between two propositions a and b.

a	b	$a \to b$
0	0	1
0	1	1
1	0	0
1	1	1

1) In proposition a→b, a is the antecedent, hypothesis or premise (sufficient condition); while, b is the consequent, conclusion or thesis (necessary condition).

2) The proposition a→b will be false only when the truth value of the antecedent is true and the truth value of the consequent is false; in any other case a→b is true.

3) The conditional connector can be found with the following grammatical terms: expressions denoting cause and effect, such as "then, "implies", "only if", "only if", "on condition that", "when", "since", "because of", "provided that", "unless", and "whenever you want that".

4) In natural language, there may be some expressions to indicate the conditional a→b between propositions a and b, such as:

"if a, then b", "a implies b", "a only if b", "a only if b", "b if a", "if a, b", "b with the condition that a", "b when a", "b since a", "b since a", "b because a", "you have b if you have a", "when a, b", "b because of", "b whenever a", "not a unless b", "b whenever you want that a".

TASK 1. THE FOLLOWING ARE NATURAL LANGUAGE EXPRESSIONS THAT ARE CONDITIONALS OF PROPOSITIONS.

1) If we have propositions a: "Ramon receives the loan" and b: "Ramon buys the house". The conditional between a and b is:

a→b: "if Ramón receives the loan, then he buys the house".

Paraphrasing this conditional, one has:

"a implies b": "

Ramon receives the loan implies that he buys the house".

b) "a only if b": "Ramon receives the loan only if he buys the house".

c) "a only if b": "Ramon receives the loan only if he buys the house".

d) "b if a": "Ramon buys the house if he receives the loan".

e) "if a, b": "If Ramon receives the loan, he buys the house".

f) "b on condition that a": "Ramon buys the house on condition that he receives the loan".

g) "b when a": "Ramon buys the house when he receives the loan".

h) "b since a": "Ramon buys the house since he receives the loan".

i) "b because a": "Ramon buys the house because he receives the loan.

j) "you have b if you have a": "you have that Ramon buys the house if you have that he receives the loan".

k) "when a, b": "when Ramón receives the loan, he buys the house".

l) "b because of": "Ramon buys the house because he receives the loan".

m) "b whenever a": "Ramon buys the house whenever he receives the loan".

n) "Not a unless b": "Ramon, he does not receive the loan unless he buys the house".

o) "b whenever you want a": "Ramón must buy the house, whenever he wants to receive the loan".

It is noted that this proposition, a→b: "if Ramon receives the loan, then he buys the house", will not be true when Ramon receives the loan and does not buy the house, that is, a: "Ramon receives the loan" has truth value and b: "Ramon buys the house" has false truth value.

TASK 2. ANALIZE THE NEXT SENTENCES

Ray tells "If the perimeter of a rectangle is 14, then its area is 10."

Which of the following could be the counterexamples? Justify your decision.

a) A rectangle with sides measuring 2 and 5

b) A rectangle with sides measuring 10 and 1

c) A rectangle with sides measuring 1 and 5

d) A rectangle with sides measuring 4 and 3

Joe examined the set of numbers {16, 27, 24} to check if they are the multiples of 3. He claimed that they are divisible by 9. Do you agree or disagree? Justify your answer.

Write the converse, inverse, and contrapositive statement for the following conditional statement.

If you study well, then you will pass the exam.

If it rains, you will get wet.

If Sally is late again I will be mad.

If you don't hurry, you will miss the bus.

If I have time, I'll finish that letter.

What will you do if you miss the plane?

Nobody will notice if you make a mistake.

If you drop that glass, it will break.

If you don't drop the gun, I'll shoot!

If you don't leave, I'll call the police.

LESSON 11 CONDITIONALS: BICONDITIONALS

Biconditional:		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

TASK 1. READ BICONDITIONAL, WRITE YOUR OUWN DEFINITION AND GIVE AN EXAMPLE.

There are propositions that are valid in both senses of the implication, which in mathematics is learned as the double implication or the "if and only if".

Let a and b be two propositions. The biconditional (or double implication) between a and b, represented symbolically by $a \leftrightarrow b$, is a new proposition that possesses truth value true if both a and b possess the same truth value; whereas, in the other two cases the truth value is false.

The connector $\leftrightarrow$ is read "if and only if". Therefore, $a \leftrightarrow b$ is read "a if and only if b".

The truth table for the biconditional between propositions a and b is given by:

Truth table for the biconditional between two propositions a and b.

a	b	$a \leftrightarrow b$
0	**0**	**1**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

1) The proposition a↔b states that it will be true only when, both a and b possess the same truth value, in any other case a↔b possesses a false truth value, that is, it will be false when the truth values of both propositions are different.

2) We often write "if" instead of writing "if and only if".

3) The biconditional connector can be found with the following grammatical terms: "if and only if", "if and only if", "when and only when" and "it is necessary and sufficient for".

4) In natural language, the biconditional proposition a↔b can be expressed with the following grammatical terms: 'a if and only if b', 'a if and only if b', 'a implies b and b implies a', 'a when and only when b' and 'a is necessary and sufficient for b'.

TASK 2. COMPLETE THE EXERCISES

Suppose this statement is true: “The garbage truck comes down my street if and only if it is Thursday morning.” Which of the following statements could be true?

a. It is noon on Thursday and the garbage truck did not come down my street this morning.

b. It is Monday and the garbage truck is coming down my street.

c. It is Wednesday at 11:59PM and the garbage truck did not come down my street today.

TASK 3. WRITE THE SOLUTION

EXERCISE 1

Given:	$a: x + 2 = 7$	
	$b: x = 5$	
Problem:	Write $a \leftrightarrow b$ as a sentence. Then determine its truth values $a \leftrightarrow b$.	

Solution: The biconditonal ……………………………………………………………..

EXERCISE 2

Given:	x: I am breathing
	y: I am alive
Problem:	Write $x \leftrightarrow y$ as a sentence.

Solution: ……
………………………………………………………………………

EXERCISE 3

Given:	r: You passed the exam.
	s: You scored 65% or higher.
Problem:	Write $r \leftrightarrow s$ as a sentence.

Solution……………………………………………………………………………………………

EXERCISE 4 : Rewrite each of the following sentences using "iff" instead of "if and only if."

if and only if	**iff**
A polygon is a triangle if and only if it has exactly 3 sides.	A polygon is a triangle iff it has exactly 3 sides.
I am breathing if and only if I am alive.	I am breathing iff I am alive.
x + 2 = 7 if and only if x = 5.	x + 2 = 7 iff x = 5.
You passed the exam if and only if you scored 65% or higher.	You passed the exam iff you scored 65% or higher.

EXPLANATION

EXERCISE 5 : GIVEN

r: A triangle is isosceles.

s: A triangle has two congruent (equal) sides.

Problem:

Is this statement biconditional? "A triangle is isosceles if and only if it has two congruent (equal) sides."

SOLUTION:

THE FOLLOWING ARE BICONDITIONAL PROPOSITIONS.

1) Given the propositions a: "a triangle is equilateral" and b: "a triangle is equiangular". The biconditional between a and b is:

a↔b: "a triangle is equilateral if and only if it is equiangular".

2) Given the propositions c: "Manuel and Maria are going to the movies" and d: "it is not raining". The biconditional between c and d is:

c↔d: "Manuel and Maria go to the movies if and only if it is not raining".

3) Given the propositions e: "today is Tuesday" and f: "yesterday was Monday". The biconditional between e and f is:

c↔d: "today is Tuesday if and only if yesterday was Monday".

TASK 4. COMPLETE THE EXERCISES

- A friend tells you "If you upload that picture to Facebook, you'll lose your job." Under what conditions can you say that your friend was wrong?

There are four possible outcomes:

1) You upload the picture and lose your job

2) You upload the picture and don't lose your job

3) You don't upload the picture and lose your job

4) You don't upload the picture and don't lose your job

- Suppose you order a team jersey online on Tuesday and want to receive it by Friday so you can wear it to Saturday's game. The website says that if you pay for expedited shipping, you will receive the jersey by Friday. In what situation is the website telling a lie?

There are four possible outcomes:

1) You pay for expedited shipping and receive the jersey by Friday

2) You pay for expedited shipping and don't receive the jersey by Friday

3) You don't pay for expedited shipping and receive the jersey by Friday

4) You don't pay for expedited shipping and don't receive the jersey by Friday

- A friend tells you "If you upload that picture to Facebook, you'll lose your job." Under what conditions can you say that your friend was wrong?

There are four possible outcomes:

1) You upload the picture and lose your job

2) You upload the picture and don't lose your job

3) You don't upload the picture and lose your job

4) You don't upload the picture and don't lose your job

- Suppose this statement is true: "If I eat this giant cookie, then I will feel sick." Which of the following statements must also be true?

a. If I feel sick, then I ate that giant cookie.

b. If I don't eat this giant cookie, then I won't feel sick.

c. If I don't feel sick, then I didn't eat that giant cookie.

- Suppose this statement is true: "The garbage truck comes down my street if and only if it is Thursday morning." Which of the following statements could be true?

a. It is noon on Thursday and the garbage truck did not come down my street this morning.

b. It is Monday and the garbage truck is coming down my street.

c. It is Wednesday at 11:59PM and the garbage truck did not come down my street today.

- “If you microwave salmon in the staff kitchen, then I will be mad at you.” If this statement is true, which of the following statements must also be true?

If you don’t microwave salmon in the staff kitchen, then I won’t be mad at you.

If I am not mad at you, then you didn’t microwave salmon in the staff kitchen.

If I am mad at you, then you microwaved salmon in the staff kitchen.

- Suppose this statement is true: “The garbage truck comes down my street if and only if it is Thursday morning.” Which of the following statements could be true?

It is noon on Thursday and the garbage truck did not come down my street this morning.

It is Monday and the garbage truck is coming down my street.

It is Wednesday at 11:59PM and the garbage truck did not come down my street today.

- Suppose this statement is true: “I wear my running shoes if and only if I am exercising.” Determine whether each of the following statements must be true or false.

I am exercising and I am not wearing my running shoes.

I am wearing my running shoes and I am not exercising.

I am not exercising and I am not wearing my running shoes.

TASK 5. PUT THE VERB INTO THE CORRECT FIRST CONDITIONAL FORM.

1. If I _________________ (go) out tonight, I _________________ (go) to the cinema.
2. If you _________________ (get) back late, I _________________ (be) angry.
3. If we _________________ (not / see) each other tomorrow, we _________________ (see) each other next week.
4. If he _________________ (come), I _________________ (be) surprised.
5. If we _________________ (wait) here, we _________________ (be) late.
6. If we _________________ (go) on holiday this summer, we _________________ (go) to Spain.

7. If the weather _________________ (not / improve), we _________________ (not / have) a picnic.

8. If I _________________ (not / go) to bed early, I _________________ (be) tired tomorrow.

9. If we _________________ (eat) all this cake, we _________________ (feel) sick.

10. If you _________________ (not / want) to go out, I _________________ (cook)

LESSON 12: SIMPLE AND COMPOUND PROPOSITIONS

Simple and Compound Propositions

a. **Simple Proposition**

A proposition that conveys one thought with no connecting words.

b. **Compound Proposition**

It contains two or more simple propositions that are put together using connective words (not, and, or, if-then, etc).

TASK 1. READ SIMPLE PROPOSITIONS AND COMPLETE THE SENTENCES

Subdivided Have are can

Simple propositions

Simple (or atomic) propositions ________________those that do not ________________logical connectors such that they ________________be________________into other propositions.

The following are simple propositions.

1) a: "Ramon bought a house".

2) b: "Maria bought a car".

3) c: "2^4 is less than 20".

TASK 2. READ THE SENTENCES AND UNDERLINE THE SIMPLE PREPOSITIONS

- Is the sun shining?
- Work Problems 4 and 5 at the end of Section 3.
- Turn in your paper now!
- 1. Houston is located in Harris County.
- San Antonio is the capital of Texas.
- Stop at the stop sign.
- When will class be over?

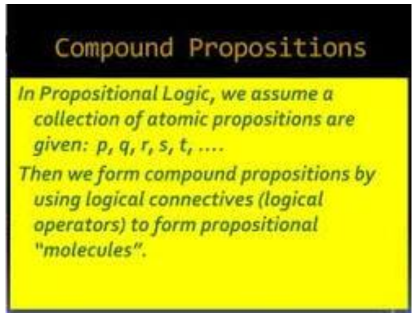
TASK 3. WRITE 10 SIMPLE PROPOSITIONS

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

TASK 4. UNDERLINE THE SIMPLE SENTENCES IN EACH COMPOUND SENTENCE BELOW.

1. Ms. Tory held Margaret's hand, but she did not speak.
2. Maizon kept Margaret from doing things, but now Maizon is gone.
3. Margaret will try new things, or she will stay the same.
4. Margaret's dad died, and she lost her best friend.
5. The summer had brought sadness, and Margaret had suffered.
6. Next summer might be better, or it might be worse.
7. Margaret hoped for better times, but she couldn't count on them.

COMPOUND PROPOSITIONS



Compound statement is made up of two or more statements. The statements are combined using words such as 'and', 'or', 'if then', 'if and only if' to form a compound statement. These words used to connect each of the individual statements to form a compound statement are called connectives. Each statement of the compound statement is called a component statement.

The following are compound propositions.

a) a: "Ramon bought a house or bought a car".

The proposition a: "Ramon bought a house or bought a car" can be decomposed into

a_1: "Ramon bought a house",

a_2: "Ramon bought a car" and the connector "disjunction".

b) b: "21 is an odd number and is divisible by 3".

The proposition b: "21 is an odd number and is divisible by 3" can be decomposed into

b_1: "21 is an odd number",

b_2: "21 is a number divisible by 3" and the connector "conjunction".

c) c: "".

TASK 1. CHECK THE NEXT COMPOUND SENTENCES

- Which of the following connective is used to form a compound statement?

For

Hence

Because

And

- What is the connective used to form the conditional compound statement?

if then

and

or

if and only if

TASK 2. USE FOR, AND, NOR, BUT, OR, YET, SO TO WRITE ONE COMPOUND SENTENCE USING THE TWO SIMPLE SENTENCES.

- Peter drove to visit his friend. They went out for dinner.

- Show a sequence of events

- Mary thinks she should go to school. She wants to get qualifications for a new profession.

- Provide a reason

- Alan invested a lot of money in the business. The business went bankrupt.

- Show an unexpected result

- Doug didn't understand the homework assignment. He asked the teacher for help.

- Show an action taken based on a reason

- The students didn't prepare for the test. They didn't realize how important the test was.

- Give a reason

- Susan thinks she should stay home and relax. She also thinks she should go on vacation.

- Show additional information

- The doctors looked at the x-rays. They decided to operate on the patient.

- Show an action taken based on a reason

- We went out on the town. We came home late.

- Show a sequence of events

- Jack flew to London to visit his Uncle. He also wanted to visit the National Museum.

- Show addition

- It is sunny. It is very cold.

- Show a contrast

- Henry studied very hard for the test. He passed with high marks.

- Provide a reason

- I would like to play tennis today. If I don't play tennis, I would like to play golf.

- Give a choice

- We needed some food for the week. We went to the supermarket.

- Show an action taken based on a reason

- Tom asked his teacher for help. He also asked his parents for help.

- Show addition

- Janet doesn't like sushi. She doesn't like any kind of fish.

- Show that Susan doesn't like either sushi or fish

TASK 3. IDENTIFY EACH SENTENCE AS EITHER SIMPLE OR COMPOUND. UNDERLINE THE SIMPLE AND CIRCLE THE COMPOUND

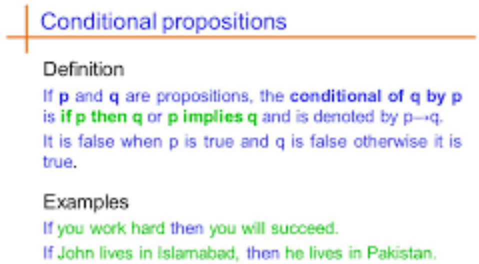
1. Maizon will attend a new school soon.
2. Margaret and Maizon have been friends for a very long time.
3. Maizon is going to Blue Hill, but Margaret will stay behind.
4. She will leave soon, and she still has to pack.
5. This last summer with Maizon is a time of great change for the girls' friendship.
6. Maizon thinks of Margaret as her best friend in the whole world.
7. Sometimes things change, and they can't change back again.
8. The friendship may end, or it may stay the same.

TASK 4. EXPLAIN WHY EACH COMPOUND SENTENCE IS INCORRECTLY WRITTEN:

a) I wanted to visit my cousin, I didn't have enough money for a bus ticket.

b) My dog lost her favourite toy, I bought her a new one.

LESSON 13: VARIATIONS OF THE CONDITIONAL OF PROPOSITIONS



Different variations of a conditional proposition: the reciprocal, inverse and counter-reciprocal.

Compound propositions having the form "if a, then b" are called conditional propositions, and are denoted symbolically by a ⟶ b.

Within this context, there are other propositions related to a conditional proposition a⟶b, which are known by: reciprocal, inverse, and counter-reciprocal (or contrapositive).

TASK 1. READ AND MATCH THE DEFINITION WITH THE CORREST WORD

CONTRAPOSITIVE	*To form the ………….. of the conditional statement, interchange the hypothesis and the conclusion.* *The …………of "If it rains, then they cancel school" is "If they cancel school, then it rains."*
CONVERSE	To form the …………of the conditional statement, take the negation of both the hypothesis and the conclusion. The …………… of “If it rains, then they cancel school” is “If it does not rain, then they do not cancel school.”

INVERSE/RECIPROCAL	To form the ……………… of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement. The …………………. of "If it rains, then they cancel school" is "If they do not cancel school, then it does not rain."

RECIPROCAL PROPOSITION

Let a and b be two propositions. The reciprocal of the conditional proposition a→b is a new proposition represented symbolically by: b→a.

THE FOLLOWING ARE RECIPROCALS OF CONDITIONAL PROPOSITIONS.

1) Consider proposition a→b: "if it is a telephone, then it is a telecommunication device". The reciprocal proposition is b→a: "if it is a telecommunication device, then it is a telephone".

2) Consider proposition c→d: "if it is raining, then the sky is cloudy". The reciprocal proposition is d→c: "if the sky is cloudy, then it is raining".

3) Consider the proposition: e→f: "if a number is divisible by 10, then it is divisible by 5". The reciprocal proposition is f→e: "if a number is divisible by 5, then it is divisible by 10".

TASK 2. COMPLETE THE ACTIVITIES

What is the inverse statement of the following conditional statement? If it is snowing, then it is cold.

If it is not snowing, then it is cold.

If it is not snowing, then it is not cold.

If it is cold, then it might be snowing.

If it is cold, then it is not warm.

1) If the wax is heated, then the wax hardens.

2) If Isabella was born in Riobamba then she is Ecuadorian.

3) If the weather is nice, I will wash the car.

4) If I read for many hours, then I will get a headache.

5) If he doesn't come back soon, then I will have to go look for him.

6) If the temperature drops to 0°C then the water will freeze.

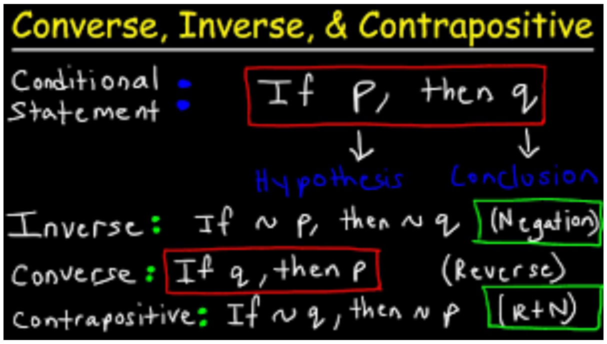
7) If a triangle has three congruent sides, it is an equilateral triangle.

8) If looks killed, then I would be dead.

9) If Mary works overtime, then she gets paid an extra half day.

10) If there is oxygen and there is spark, then paper burns.

INVERSE PROPOSITION



Let a and b be two propositions. The inverse of the conditional proposition a→b is a new proposition represented symbolically by: ¬a⟶¬b.

The following are inverses of conditional propositions.

1) Consider proposition a⟶b: "if it is a telephone, then it is a telecommunication device". The inverse proposition is ¬a⟶¬b: "if it is not a telephone, then it is not a telecommunication device".

2) Consider proposition c⟶d: "if it is raining, then the sky is cloudy". The converse proposition is ¬c⟶¬d: "if it is not raining, then the sky is not cloudy".

3) Consider the proposition: e⟶f: "if a number is divisible by 10, then it is divisible by 5". The converse proposition is ¬e⟶¬f : "If a number is not divisible by 10, then it is not divisible by 5".

TASK 3. COMPLETE THE EXERCISES

What is the inverse statement of the following conditional statement? If it is snowing, then it is cold.

If it is not snowing, then it is cold.

If it is not snowing, then it is not cold.

If it is cold, then it might be snowing.

If it is cold, then it is not warm.

1) If I get paid today, then I'm going to the movies.

2) If Alejandro is hungry, then he makes himself a sandwich.

3) If the sun comes out tomorrow, then we go to the zoo.

4) If the math teacher takes more than two hours talking, then the students fall asleep.

5) If Maria is idle, then Ramon went to visit her.

6) I'll tell you a secret if you promise to keep quiet.

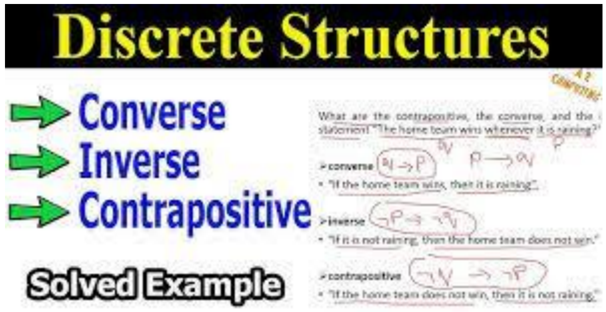
7) If the math program is readable, then it is well structured.

8) If I have time, then we have lunch together.

9) If Manuel leaves his house early, then we are on time for the soccer game.

10) If Alejandra has 200 credits accumulated in the math program, then she graduates.

CONTRAPOSITIVE -PROPOSITION



The counter reciprocal (or contrapositive) of the conditional proposition $a \rightarrow b$ is a new proposition represented symbolically by: $\neg b \longrightarrow \neg a$.

The following are counter-reciprocals of conditional propositions.

1) Consider proposition $a \longrightarrow b$: "if it is a telephone, then it is a telecommunication device". The counterreciprocal proposition is $\neg b \longrightarrow \neg a$: "if it is not a telecommunication device, then it is not a telephone".

2) Consider proposition $c \longrightarrow d$: "if it is raining, then the sky is cloudy". The counter proposition is $\neg d \longrightarrow \neg c$: "if the sky is not cloudy, then it is not raining".

3) Consider the proposition: $e \longrightarrow f$: "if a number is divisible by 10, then it is divisible by 5". The counter-proposition is $\neg f \longrightarrow \neg e$: "if a number is not divisible by 5, then it is not divisible by 10".

TASK 4. SELECT THE STATEMENT THAT IS THE CONVERSE OF "IF I HAD A HAMMER, I WOULD HAMMER IN THE MORNING."

A. If I didn't have a hammer, I wouldn't hammer in the morning."

B. If I don't hammer in the morning, I don't have a hammer.

C. If I hammer in the morning, I have a hammer.

D. If I had a ham, I would eat ham in the morning.

SELECT THE STATEMENT THAT IS THE INVERSE OF "IF IT RAINS, THEN I WON'T GO TO CLASS."

A. If I don't go to class, then it rains.

B. If it doesn't rain, then I will go to class.

C. If I go to class, then it isn't raining.

D. Since it's Friday I probably won't go to class, anyway.

TASK 5. WHAT IS THE CONTRAPOSITIVE STATEMENT FOR THE FOLLOWING CONDITIONAL STATEMENT?

- "If it rains, then they cancel school."

The contrapositive of "If it rains, then they cancel school" is "If they do not cancel school, then it does not rain."

- If two angles do not have the same measure, then they are not congruent.

The contrapositive

- If a quadrilateral is a rectangle, then it has two pairs of parallel sides.

The contrapositive

- The reciprocal of any irrational number is irrational.

The contrapositive

- If it is raining, then the grass is wet.

The contrapositive

- If two angles have the same measure, then the two angles are congruent.

The contrapositive

- If all figures are four-sided planes, then figures are rectangles.

The contrapositive

- If it is a triangle, then it is a polygon.

The contrapositive

- All roses are flowers. So all non-flowers are non-roses.

The contrapositive

- If a number is a multiple of 8, then the number is a multiple of 4.

The contrapositive

- If it rained last night, then the sidewalk is wet

The contrapositive

TASK 6 COMPLETE THE EXERCISE

The conditional statement would be: "If all figures are four-sided planes, then figures are rectangles."

Converse: ____

Inverse: "…………………………………………………………………………...."

Contrapositive: "……………………………………………………………………..."

TASK 7. COMPLETE THE TABLES

Statement	If two angles are congruent, then they have the same measure.
Converse	
Inverse	
Contrapositive	

Statement	If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
Converse	
Inverse	
Contrapositive	

Statement	If you earned a bachelor's degree, then you got a high-paying job.

Converse	
Inverse	
Contrapositive	

Statement	If you do well in math classes, then you are smart.
Converse	
Inverse	
Contrapositive	

Statement	If your blood type is type , then you are classified as a universal blood donor.
Converse	

Inverse	
Contrapositive	

Statement	If we make a mistake on the dosage, that patient may not survive
Converse	
Inverse	
Contrapositive	

TRUTH VALUES FOR VARIATIONS OF THE CONDITIONAL OF PROPOSITIONS

Truth value of a conditional statement		
p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

A conditional can be true, but not necessarily its variants.

Some examples to illustrate this.

In general, this is the case with implications:

If A is true, then B is true.

That this occurs does not mean either If B is true, then A is true or If A is not true, then B is not true, but in many cases it is thought to be true.

I will give a simple example, which is the one I use most often with my students

"IF SNOW FALLS THEN IT IS COLD". This does not mean that if it is cold then snow falls, because the temperature can drop without snow; so a sufficient condition for it to be cold is that snow falls, but this is not a necessary condition.

IF I AM A GRADUATE OF A CAREER THEN I AM AN ALUMNUS OF THAT CAREER; that does not mean that because I am an alumnus then I am a graduate, since I could have dropped out without graduating. The implication is only valid in one sense: "graduate ⇒ alumnus", but it is not correct to extend it to: "alumnus ⇒ graduate", which seems to be the implication used by some to speak of their non-existent degrees.

TASK 8. COMPLETE THE EXERCISES

1) Consider the following conditional proposition: "if it rains, then my yard gets wet".

DISCUSS THE TRUTH VALUES OF THE VARIATIONS OF THIS CONDITIONAL.

2) Consider the following conditional proposition: "if snow falls, then it is cold".

IDENTIFY THE NECESSARY CONDITION.

3) Consider the following conditional proposition to be true: "if you do not pay, you will be fined".

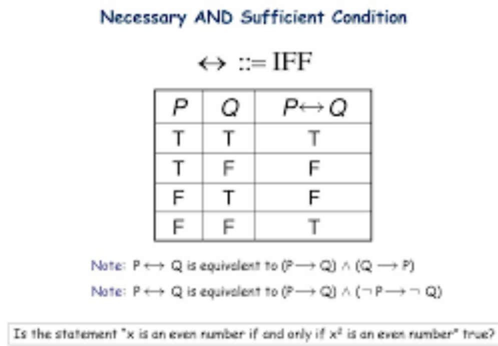
8) Consider the following situation: a girl tells her boyfriend "if I don't get a job, then I won't marry you".

LESSON 14: NECESSARY AND/OR SUFFICIENT CONDITIONS OF A PROPOSITION

To the statement of a given conditional proposition, the notions of necessary condition and sufficient condition are associated. These elements will be the focus of interest in the present lesson. Especially, the meaning of necessary and sufficient conditions, as well as the conditions that are necessary and/or sufficient; that is, the logical meaning of a conditional or biconditional. Where, its importance increases precisely at the moment of approaching propositions in mathematics, since, knowing how to identify which are the necessary and sufficient conditions allows one to determine whether one is in the presence of an implication or a double implication, popularly known in mathematics as "the then,⇒" and the "if and only if, ⇔", respectively.

We begin by pointing out that, the distinction between a necessary condition and a sufficient condition of a proposition, is something that many students are unaware of, do not apply, confuse, or misuse. Although, using the definition of each of these conditions separately, it is not difficult to identify them, in practice it is not so for the analysis of a condition, since there are many circumstances in which learners find it difficult to distinguish between the two. That is, it is common to confuse a necessary condition as a sufficient condition, and vice versa; or also, if they have a condition that is only necessary (or only sufficient), to think that it is both necessary and sufficient.

NECESSARY CONDITION OF A PROPOSITION



Necessary AND Sufficient Condition

$\leftrightarrow$ ::= IFF

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Note: $P \leftrightarrow Q$ is equivalent to $(P \longrightarrow Q) \wedge (Q \longrightarrow P)$

Note: $P \leftrightarrow Q$ is equivalent to $(P \longrightarrow Q) \wedge (\neg P \longrightarrow \neg Q)$

Is the statement "x is an even number if and only if x^2 is an even number" true?

If a and b are two propositions such that one has the conditional a→b, then b is said to be a necessary condition for a.

Thus, a necessary condition is a conclusion reached as a result of some specific event. In other words, if a fact exists, then, necessarily, there must also exist an event that is a conclusion.

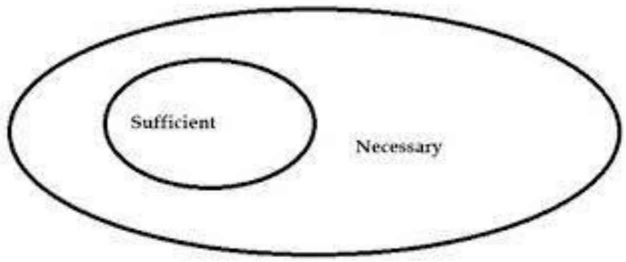
TASK 1. IDENTIFY THE NECESSARY CONDITION OF A PROPOSITION.

- Consider the following conditional proposition: "if it rains, then my yard gets wet". Identify the necessary condition for this proposition.

- Consider the following conditional proposition: "if snow falls, then it is cold". Identify the necessary condition for this proposition.

- Consider the following conditional proposition: "if n is a multiple of 4, then n is a multiple of 2", for all n natural numbers. What is the necessary condition for this proposition?
-

SUFFICIENT CONDITION OF A PROPOSITION



If a and b are two propositions such that one has the conditional a→b, a is said to be a sufficient condition for b.

Thus, a proposition (a) is a sufficient condition for another proposition (b) if the truth of the first (antecedent) would guarantee the truth of the second (consequent). That is, the truth of the first proposition is sufficient -all that is needed- to ensure the truth of the second proposition.

TASK 2. IDENTIFYING THE SUFFICIENT CONDITION OF A PROPOSITION.

1) Consider the following conditional proposition: "if it rains, then my yard gets wet". Identify the sufficient condition for this proposition.

2) Consider the following conditional proposition: "if snow falls, then it is cold". Identify the sufficient condition for this proposition.

3) Consider the following conditional proposition: "if n is a multiple of 4, then n is a multiple of 2", for all n natural numbers. What is the sufficient condition for this proposition?

When the conditional proposition a→b is true, it can be paraphrased as follows: "a is enough for b" and "b on the condition that a".

When dealing with conditional propositions, especially when formally studying mathematics, it is relevant to know how to identify which of the conditions involved is necessary and which is sufficient, since this will help with the correct use of existing results and the derivation of others.

TASK 3. PARAPHRASE OF A PROPOSITION USING THE NECESSARY AND SUFFICIENT CONDITION .

- Consider that the following proposition: "if n is divisible by 16, then n is divisible by 2", is true. Paraphrase the proposition using the necessary and sufficient condition.

Solution.

- Consider the following true conditional proposition: "if Ramon receives the loan, then he buys the house".

State the necessary and sufficient conditions for this proposition.

Solution.

The proposition can be written in the form: $a \rightarrow b$, where a: "Ramón receives the loan" and b: "he buys the house".

a) Necessary condition:

b) Sufficient condition:

2) Consider the following true conditional proposition: "if Isabela is pregnant, then she had sexual intercourse". State the necessary and sufficient conditions for this proposition.

Solution.

a)Necessary condition:

b) Sufficient condition.

3) Suppose that the following proposition: "I will accept the loan on condition that the interest rates are not high", is true. State the necessary and sufficient conditions for this proposition.

Solution.

4) Consider that the following conditional proposition: "if you do not pay, you will be fined", is true. State the necessary and sufficient conditions for this proposition.

Solution.

5) Consider the following conditional proposition: "Alejandro goes to the beach only if Maria accompanies him". Interpret the proposition in terms of necessary and sufficient conditions.

Solution.

CONDITION ONLY NECESSARY, ONLY SUFFICIENT OR, NECESSARY AND SUFFICIENT

$$\frac{\partial f(\mathbf{X}_0)}{\partial x_j} < 0 \text{ or } \frac{\partial f(\mathbf{X}_0)}{\partial x_j} > 0$$

By selecting h_j with appropriate sign, it is always possible to have

$$h_j \frac{\partial f(\mathbf{X}_0)}{\partial x_j} < 0$$

Setting all other h_j equal to zero, Taylor's expansion yields

$$f(\mathbf{X}_0 + \mathbf{h}) < f(\mathbf{X}_0)$$

It is common among the student community, given a conditional proposition, to verify whether it is only conditional or it is possible that it is also considered a biconditional. That is, confusing whether a condition is only necessary or only sufficient, or whether it is both necessary and sufficient.

TASK 4. CREATE YOUR OWN EXAMPLES OF CONDITIONAL STATEMENTS THAT SATISFY THE FOUR COMBINATIONS OF NECESSITY AND SUFFICIENCY

1. Necessary but not sufficient

2. Sufficient but not necessary

3. Both necessary and sufficient

4. Neither necessary nor sufficient

1) If everyone agrees, then Carlos brings the drink.

2) You will cut yourself if you take the knife that way.

3) If the water boils, add the pasta.

4) If you are thirsty, pour yourself some water.

5) If you study harder, then you get better grades.

6) If you stay for coffee, then I make a cake.

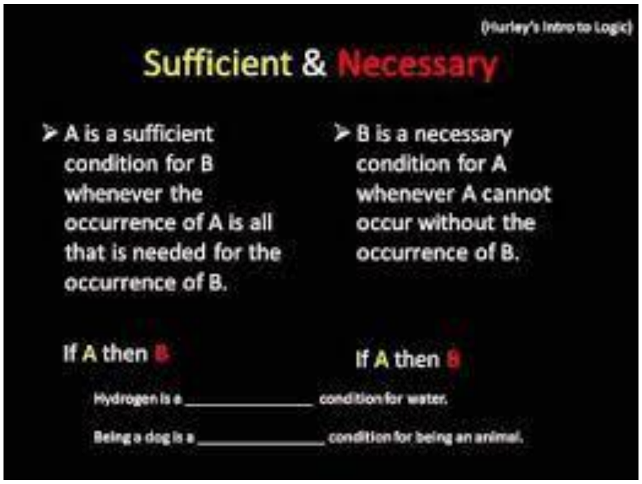
7) I help you if you tidy up your room.

8) If you are hot, we open the window.

9) I'll study for the exam if you don't need my help anymore.

10) If your arm hurts, then put ice on it.

CONDITION ONLY NECESSARY OR ONLY SUFFICIENT



When one deals with conditions within a conditional proposition that are only necessary or only sufficient, one is in the presence of a conditional proposition without the possibility of being considered a biconditional.

TASK 5. COMPLETE THE EXERCISES

1) Consider the following conditional proposition: "if it rains, then my yard gets wet". Analyze the conditions: a) "it rains" and b) "my yard gets wet"; to identify if they are necessary and/or sufficient.

Solution.

The condition

The condition

2) Consider the following conditional proposition: "if snow falls, then it is cold". Analyze the conditions: a) "snow falls" and b) "it is cold"; to identify if they are necessary and/or sufficient.

a) The condition

b) The condition

3) Consider the following conditional proposition: "if n is a multiple of 4, then n is a multiple of 2", for all n natural numbers. Analyze the conditions: a) "n is a multiple of 4" and b) "n is a multiple of 2"; to identify if they are necessary and/or sufficient.

Solution.

The condition

The condition

4) **Consider that the following conditional proposition: "if Alejandro passes the subject of mathematical logic, then I study", is true. Analyze the conditions: a) "Alejandro passes the subject of mathematical logic" and b) "I study"; to identify if they are necessary and/or sufficient.**

Solution.

a) The condition

b) The condition

1) Analyze the conditions: a: "you do not hand in the material on time" and b: "they will take away five points"; to identify if they are necessary and/or sufficient.

2) Analyze the conditions: a: "you interrupt the teacher again" and b: "the teacher will get annoyed"; to identify if they are necessary and/or sufficient.

3) Analyze the conditions: a: "let the boss know in time" and b: "you can take a day off"; to identify if they are necessary and/or sufficient.

4) Analyze conditions: a: "you promise to be on time" and b: "I'll take you to the dance"; to identify if they are necessary and/or sufficient.

5) Analyze conditions: a: "pay attention to everything the English teacher says" and b: "you will learn English sooner"; to identify if they are necessary and/or sufficient.

6) Analyze conditions: a: "these packages are out of your budget" and b: "we have cheaper packages"; to identify if they are necessary and/or sufficient.

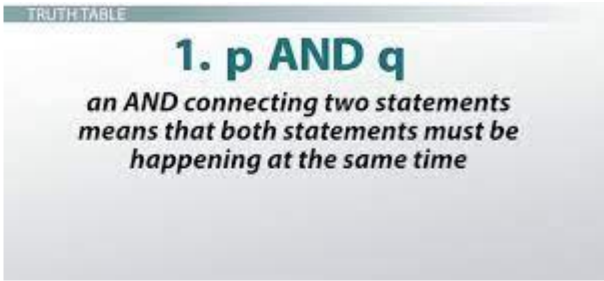
7) Analyze the conditions: a: "you will accompany me" and b: "I would be very happy"; to identify if they are necessary and/or sufficient.

8) Analyze the conditions: a: "I attend class in the afternoons" and b: "I could advance the contents"; to identify if they are necessary and/or sufficient.

9) Analyze the conditions: a: "I tell you the truth" and b: "you cannot tell anyone else"; to identify if they are necessary and/or sufficient.

10) Analyze the conditions: a: "Manuel has the luggage ready" and b: "they call Manuel for the trip"; to identify if they are necessary and/or sufficient.

A PROPOSITION THAT IS A SUFFICIENT CONDITION FOR TWO PROPOSITIONS.



TASK 6. COMPLETE THE EXERCISES

1) Consider the following propositions, a: "the triangle is equilateral", b_1: "the triangle is equiangular" and b_2: "the triangle has all equal sides". Verify that proposition a is a sufficient condition for propositions b_1 and b_2.

Solution.

2) Consider the following propositions, a_1: "the plane quadrilateral has its sides congruent", a_2:" the plane quadrilateral has the four right interior angles" and b: "the quadrilateral is a square". Verify that propositions a_1 and a_2 are sufficient conditions for proposition b.

Solution.

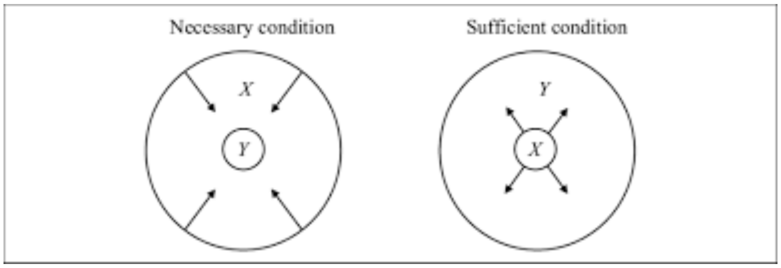
■

A proposition that is a necessary condition for two propositions.

1) Consider the following propositions, a_1: "the triangle has the angles of the base equal and the opposite different" and a_2: "the triangle has two equal sides and the one of the base different" and b: "the triangle is isosceles". Verify that proposition b is a necessary condition for propositions a_1 and a_2.

Solution.

NECESSARY AND SUFFICIENT CONDITIONS AT THE SAME TIME



When you address conditions within a conditional proposition that are both necessary and sufficient, you are in the presence of a biconditional proposition.

TASK 7. COMPLETE THE EXERCISES : NECESSARY AND SUFFICIENT CONDITIONS AT THE SAME TIME.

1) Consider the following conditional proposition: "if a triangle is equilateral, then it is equiangular". Analyze the conditions: a) "a triangle is equilateral" and b) "it is equiangular"; to identify if they are necessary and/or sufficient.

Solution.

i.The condition

ii. The condition

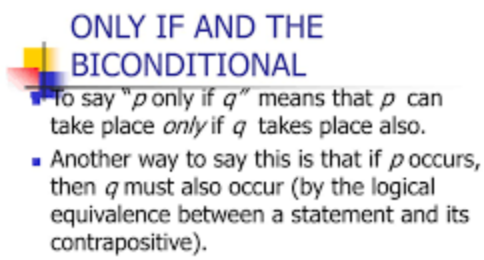
2) Consider the following conditional proposition: "if today is Tuesday, then yesterday was Monday". Analyze the conditions: a) "today is Tuesday" and b) "yesterday was Monday"; to identify if they are necessary and/or sufficient.

Solution.

a) The condition

b) The condition

EQUIVALENCE BETWEEN THE EXPRESSIONS "BICONDITIONAL", "NECESSARY AND SUFFICIENT CONDITIONS" AND "IF, AND ONLY IF".



If we consider the expression "a if, and only if, b" for two propositions a and b, we have the conjunction of two expressions, namely, "a, if b" and "a, only if b".

On the one hand, "a, if b" says that if b occurs then a occurs, that is, b is sufficient condition for a. Thus, this expression has the same meaning as the conditional "if b, then a", which is symbolically written by b→a.

While, on the other hand, "a, only if b" says that a occurs only if b occurs, i.e., the proposition b is a necessary condition for a to occur. In other words, whenever a is fulfilled, necessarily b is also fulfilled. Thus, this expression has the same meaning as the conditional "if a, then b", which is written symbolically by a→b.

Therefore, the combining of both of the above cases, give rise to the expression "if, and only if" is equivalent "a ↔ b". Concretely, the meaning of a↔b is the same as the conjunction of a→b and a→b.

TASK 8. VERIFY THE NECESSARY AND SUFFICIENT CONDITIONS OF A BICONDITIONAL.

1) Consider the following conditional proposition: "if a triangle is equilateral, then it is equiangular". From necessary and/or sufficient conditions verify whether it can be stated that "a triangle is equilateral if, and only if, it is equiangular".

Solution**.**

2) Consider the following conditional proposition: "if today is Tuesday, then yesterday was Monday". Analyze the conditions: a) "today is Tuesday" and b) "yesterday was Monday"; to identify if they are necessary and/or sufficient.

Solution.

- Consider the conditions: a: "David teaches mathematics" and b: "David is a mathematics teacher", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Mary buys the car" and b: "Mary receives the loan", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "John bought the movie ticket" and b: "John has the right to enter the movie theater", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Sofia votes in the elections" and b: "Sofia is registered in the electoral registry", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Alejandro lives in Ecuador" and b: "Alejandro speaks Spanish", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "this month is November" and b: "the previous month was October", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Christmas is only a few days away" and b: "this month is December", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "I speak English" and b: "I pass the Cambridge exam", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Lourdes is a vegetarian" and b: "Lourdes does not eat meat", identify if they are necessary and sufficient conditions.

- Consider the conditions: a: "Carlos plays the guitar in a musical group" and b: "Carlos is a musician", identify if they are necessary and sufficient conditions.

LESSON 15: TRANSLATION OF TEXTS FROM A NATURAL LANGUAGE TO A SYMBOLIC LANGUAGE OF PROPOSITIONAL LOGIC

Within the area of mathematics, one of the most used and useful strategies is the application of symbolic notation within a symbolic language according to the subject studied. This is done with the purpose of abbreviating mathematical content to facilitate its manipulation and understanding; in addition, it is a way that helps to present complex reasoning in a more simplified way. Thus, this lesson is dedicated to the translation of texts from an everyday language containing simple propositions and logical connectors to a symbolic language within the context of propositional logic.

The following is a method consisting of four steps to translate an everyday language into a symbolic one within propositional calculus. In this sense, we have:

TASK 1. READ AND ORDER THE 4 STEPS TO TRANSLATE

S_...........) Compare the text in everyday language with the expression obtained in symbolic language. This, with the purpose of verifying that in both contexts the same thing is obtained.

S_...........) Identify the logical connectors within the text that link the simple propositions already identified in the previous step.

S_..........) Identify the simple propositions involved in the text. Care must be taken not to repeat the propositions, since there may be propositions within the text that are simply the negation of another already identified.

S_...........) Perform the translation from everyday language to symbolic language.

It should be noted that this procedure is only a guide that helps to guide the translation process, therefore, it is neither universal nor definitive. It is also important to highlight that each text may involve details in its narrative form that can make a difference in the translation process if not approached with care. Specifically, the richness of natural languages allows the use of a diversity of grammatical terms to present certain situations. Thus, even if the logical expression and the text are written in different ways, they must express the same thing.

TASK 2. COMPLETE THE EXERCISES. DIFFERENT GRAMMATICAL TERMS ARE USED FOR THE SAME EXPRESSION.

1) Consider the following text written in everyday language: "if car prices are high, bank loans are useful". Write this expression in another form.

Solution.

2) Consider the following text written in everyday language: "the house is small, but comfortable". Write this expression in another form.

Solution.

TASK 3. TRANSLATION OF A TEXT FROM EVERYDAY LANGUAGE TO THE SYMBOLIC LANGUAGE OF LOGIC.

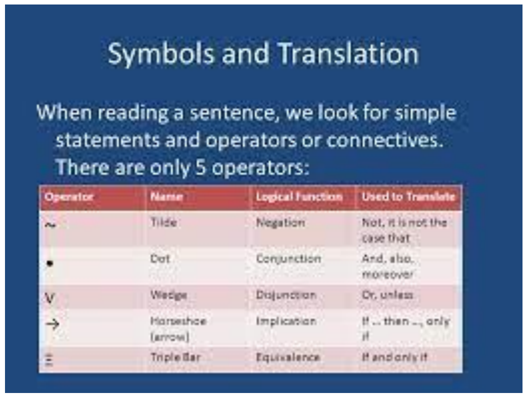
1) Consider the following text written in everyday language: "If private security is effective, assault rates decrease in the city and tourism develops. Assault rates do not decrease, but private security is effective. So, tourism does not develop. Translate the text into a symbolic language within the propositional calculus.

Solution.

2) Consider the following text written in everyday language: "When Maria left she could have gone west or east. If Maria went west, then she arrived in Manabi. Every time Maria goes to Manabi she visits Alejandro. If she went east, then she passed through Tena. When Maria passes through Tena she continues her journey to Francisco de Orellana or to Nueva Loja. Maria did not reach Francisco de Orellana and Alejandro did not see Maria. Therefore, Maria is in Nueva Loja". Translate the text into a symbolic language within propositional calculus.

Solution.

LESSON 16: TRANSLATION OF EXPRESSIONS FROM A SYMBOLIC LANGUAGE OF PROPOSITIONAL LOGIC TO A NATURAL ONE



TASK 1 READ AND COMPLETE THE TEXT BELOW

KNOWING LANGUAGES USED MATHEMATICS TRANSLATIONS
SYMBOLIC NECESSARY INVERSE MAKE NATURAL
UNDERSTANDING

The previous lesson highlighted the relevance of _________________the symbolic _________________that are _________________for the study of some topics within the area of_________________It is not enough to be able to make_________________from a natural language to a _________________one, but it is also _________________to be able to make the_________________process, that is to say, to _________________translations from a symbolic language to the learner's _________________language. This, with the purpose of_________________the mathematical contents studied, especially the reasoning involved. For example, the latter occurs when advanced topics in mathematics are studied and written by different authors. Here, unification is necessary and is crystallized through the translation from symbolic to natural language. It is in this sense that the present lesson is devoted to the translation of expressions within the context of a symbolic language of propositional logic into texts of everyday language.

As in the previous lesson, the following is a method consisting of four steps to translate a symbolic language within the propositional calculus into an everyday language.

TASK 2. ORDER THE STEPS TO TRANSLATE

S___________ Analyze the written expression in symbolic language within the propositional calculus, taking care to identify if the simple propositions are among those given in the previous step, as well as the logical connectors that link them.

S___________ Examine the given simple propositions, taking care of the respective notation assigned.

S___________ Perform the translation from symbolic language to everyday language.

Compare the written expression in symbolic language with the text obtained in everyday language. This, with the purpose of verifying that in both contexts the same thing is obtained.

Similarly, the steps presented are only a reference guide to help orient the translation process. It should also be noted that each translation involves details of care in the narration, first, through the use of grammatical terms, and second, when it is necessary to negate propositions; since this will often require certain pertinent adjustments to consolidate a coherent text within the context of a natural language. Likewise, the use of these last two aspects must be subject to the fact that both the logical expression and the text must be able to express the same thing.

TASK 3. TRANSLATION AN EXPRESSION FROM A SYMBOLIC LANGUAGE OF PROPOSITIONAL LOGIC TO A NATURAL ONE.

1. Consider the following two propositions, a: "Alexander studies a mathematics degree" and b: "Alexander likes the subject of mathematical logic". Perform the translations of the following propositions:

a) $a \rightarrow b$

b) $a \rightarrow \neg b$

c) $\neg a \vee b$

d) $a \wedge \neg b$

Solution.

- This with the purpose of identifying if the simple propositions (a and b) are found in the ones provided in the previous step; as well as, which are the logical connectors (conditional, negation, disjunction and conjunction) that operate; in order to only thus, perform the subsequent translation.

Third, the translation from symbolic language to everyday language is:

a) a→b: "if Alejandro studies a mathematics degree, then he likes the subject of mathematical logic".

b) a→¬ b: "if Alejandro studies a mathematics degree, then he does not like the subject of mathematical logic".

c) ¬ a vb: "Alejandro does not study a mathematics major or does not like the subject of mathematical logic".

d) a∧¬ b: "Alejandro studies a mathematics degree and does not like the subject of mathematical logic".

1) Consider the following two propositions a: "Alejandra lives to eat" and b: "Alejandra eats to live". Make the translations of the following propositions:

a) $a \wedge b$

b) $a \vee b$

c) $(\neg a \wedge \neg b)$

d) $(\neg a \vee \neg b)$

e) $(\neg a \vee b)$

2) Consider the following two propositions a: "I am wrong", b: "question number one is true" and c: "question number two is false". Make translations of the following propositions:

a) $a \rightarrow (b \wedge c)$

b) $(a \vee b) \wedge c$

c) $a \vee (b \wedge c)$

d) $(b \wedge c) \ \rightarrow a$

e) $a \leftrightarrow (b \wedge c)$

3) Consider the following two propositions a: "book costs more than $20" and b: "Antonio will not be able to buy it". Make the translations of the following propositions:

a) $a \rightarrow b$

b) $\neg b \rightarrow \neg a$

c) $\neg a \rightarrow \neg b$

d) $a \vee \neg b$

e) $\neg a \vee b$

4) Consider the following two propositions a: "the number on the screen is less than four", b: "the number on the screen is greater than ten" and c: "the number on the screen is not equal to six". Make the translations of the following propositions:

a) $(a \vee b) \rightarrow c$

b) $(a \vee b) \rightarrow \neg c$

c) $\neg c \rightarrow (\neg a \wedge \neg b)$

d) $(a \vee b) \leftrightarrow c$

e) $\neg(a \vee b) \leftrightarrow \neg c$

5) Consider the following two propositions a: "Nanci's cédula number is less than sixteen million" and b: "Nanci's cédula number is greater than seventeen million". Make the translations of the following propositions:

a) $a \vee b$

b) $\neg a \wedge \neg b$

c) $\neg a \vee \neg b$

d) $a \rightarrow \neg b$

e) $b \rightarrow \neg a$

6) Consider the following two propositions a: "we won the first game", b: "we won the Second game" and c: "we won the championship". Make the translations of the following propositions:

a) $(a \wedge b) \rightarrow c$

b) $\neg c \rightarrow (\neg a \vee \neg b)$

c) $(\neg a \wedge b) \rightarrow \neg c$

d) $(a \wedge \neg b) \rightarrow \neg c$

e) $(\neg a \vee b) \rightarrow \neg c$

7) Consider the following two propositions a: "roses are red" and b: "daisies are yellow". Make the translations of the following propositions:

a) $a \wedge b$

b) $a \vee b$

c) $(\neg a \wedge \neg b)$

d) $(\neg a \vee \neg b)$

e) $(\neg a \vee b)$

8) Consider the following two propositions a: "Alejandra wants to eat fruit" and b: "Alejandra does not want to eat ice cream". Make the translations of the following propositions:

a) $a \wedge b$

b) $a \vee b$

c) $(\neg a \wedge \neg b)$

d) $(\neg a \vee \neg b)$

e) $(\neg a \vee b)$

9) Consider the following two propositions a: "Alejandra is eating", b: "Alejandra is drinking" and c: "Alejandra is having fun". Make the translations of the following propositions:

a) $(a \wedge b) \rightarrow c$

b) $a \vee (b \wedge c)$

c) $(a \vee b) \wedge c$

d) $a \vee b \vee c$

e) $a \wedge b \wedge c$

10) Consider the following two propositions a: "Manuel arrives to class tenprano", b: "Maria arrives to class late" and c: "El professor se molesta". Make the translations of the following propositions:

a) $a \rightarrow \neg c$

b) $b \rightarrow c$

c) $(a \lor b) \to c$

d) $(a \land b) \to c$

e) $(\neg a \lor b) \to c$

GRAMMAR REFERENCE

IRREGULAR VERBS

Infinitive	Simple Past	Past Participle	Spanish
arise	Arose	arisen	surgir
be	was / were	been	ser
beat	Beat	beaten	golpear
become	Became	become	convertirse
begin	Began	begun	comenzar
bet	bet/betted	bet/betted	apostar
bite	Bit	bitten	morder
bleed	Bled	bled	sangrar
blow	Blew	blown	soplar
break	Broke	broken	romper
bring	Brought	brought	traer
build	Built	built	construir

buy	Bought	bought	comprar
catch	Caught	caught	atrapar
choose	Chose	chosen	elegir
come	Came	come	venir
cost	Cost	cost	costar
creep	Crept	crept	arrastrarse
cut	Cut	cut	cortar
deal	Dealt	dealt	dar, repartir
do	Did	done	hacer
draw	Drew	drawn	dibujar
dream	dreamt/dreamed	dreamt/dreamed	soñar
drink	Drank	drunk	beber
drive	Drove	driven	conducir
eat	Ate	eaten	comer
fall	Fell	fallen	caer

feed	Fed	fed	alimentar
feel	Felt	felt	sentir
fight	Fought	fought	pelear
find	Found	found	encontrar
flee	Fled	fled	huir
fly	Flew	flown	volar
forget	Forgot	forgotten	olvidar
forgive	Forgave	forgiven	perdonar
forsake	Forsook	forsaken	abandonar
freeze	Froze	frozen	congelar
get	Got	got	tener, obtener
give	Gave	given	dar
go	Went	gone	ir
grind	Ground	ground	moler
grow	Grew	grown	crecer

hang	Hung	Hung	colgar
have	Had	Had	tener
hear	Heard	Heard	oír
hide	Hid	Hidden	esconderse
hit	Hit	Hit	golpear
hold	Held	Held	tener, mantener
hurt	Hurt	Hurt	herir, doler
keep	Kept	Kept	guardar
kneel	Knelt	Knelt	arrodillarse
know	Knew	Known	saber
lead	Led	Led	encabezar
learn	learnt/learned	learnt/learned	aprender
leave	Left	Left	dejar
lend	Lent	Lent	prestar
let	Let	Let	dejar

lie	Lay	Lain	yacer
lose	Lost	Lost	perder
make	Made	Made	hacer
mean	Meant	Meant	significar
meet	Met	Met	conocer, encontrar
pay	Paid	Paid	pagar
put	Put	Put	poner
quit	quit/quitted	quit/quitted	abandonar
read	Read	Read	leer
ride	Rode	Ridden	montar, ir
ring	Rang	Rung	llamar por teléfono
rise	Rose	Risen	elevar
run	Ran	Run	correr
say	Said	Said	decir
see	Saw	Seen	ver

sell	Sold	Sold	vender
send	Sent	Sent	enviar
set	Set	Set	fijar
sew	Sewed	sewn/sewed	coser
shake	Shook	Shaken	sacudir
shine	Shone	Shone	brillar
shoot	Shot	Shot	disparar
show	Showed	shown/showed	mostrar
shrink	shrank/shrunk	Shrunk	encoger
shut	Shut	Shut	cerrar
sing	Sang	Sung	cantar
sink	Sank	Sunk	hundir
sit	Sat	Sat	sentarse
sleep	Slept	Slept	dormir
slide	Slid	Slid	deslizar

sow	Sowed	sown/sowed	sembrar
speak	Spoke	Spoken	hablar
spell	spelt/spelled	spelt/spelled	deletrear
spend	Spent	Spent	gastar
spill	spilt/spilled	spilt/spilled	derramar
split	Split	Split	partir
spoil	spoilt/spoiled	spoilt/spoiled	estropear
spread	Spread	Spread	extenderse
stand	Stood	Stood	estar de pie
steal	Stole	Stolen	robar
sting	Stung	Stung	picar
stink	stank/stunk	Stunk	apestar
strike	Struck	Struck	golpear
swear	Swore	Sworn	jurar
sweep	Swept	Swept	barrer

swim	Swam	Swum	nadar
take	Took	taken	tomar
teach	Taught	taught	enseñar
tear	Tore	torn	romper
tell	Told	told	decir
think	Thought	thought	pensar
throw	Threw	thrown	lanzar
tread	Trode	trodden/trod	pisar
understand	understood	understood	entender
wake	Woke	woken	despertarse
wear	Wore	worn	llevar puesto
weave	Wove	woven	tejer
weep	Wept	wept	llorar
win	Won	won	ganar
wring	Wrung	wrung	retorcer

write	Wrote	written	escribir

TO BE

The verb be, positive, negative, interrogative statements.

To Be - Affirmative

Subject	**To Be**	**Examples**
I	am	I **am** from New Zealand.
You	are	You **are** Chilean.
He	is	He **is** twenty years old.
She	is	She **is** a nurse.
It	Is	It **is** a big dog.
We	Are	We **are** intelligent.
You	Are	You **are** students.
They	Are	They **are** married.

To Be - Negative Sentences

The negative of **To Be** can be made by adding **not** after the verb.

Subject	To Be	Examples
I	am not	I **am not** from Spain.
You	are not	You **are not** Australian.
He	is not	He **is not** thirty years old.
She	is not	She **is not** a secretary.
It	is not	It **is not** a small cat.
We	are not	We **are not** stupid.
You	are not	You **are not** teachers.
They	are not	They **are not** single.

To Be - Questions

To create questions with **To Be**, you put the **Verb** before the **Subject**.

Affirmative	Question
I am intelligent.	**Am I** intelligent?
You are a student.	**Are you** a student?
He is a pilot.	**Is he** a pilot?
She is from Spain.	**Is she** from Spain?
It is a big house.	**Is it** a big house?
We are ready.	**Are we** ready?
You are doctors.	**Are you** doctors?
They are rich.	**Are they** rich?

To Be - Short Answers

Question	Short Answers**	Short Answers
Am I intelligent?	Yes, you are.	No, you aren't.
Are you a student?	Yes, I am.	No, I am not.
Is he a pilot?	Yes, he is.	No, he isn't.
Is she from Spain?	Yes, she is.	No, she isn't.
Is it a big house?	Yes, it is.	No, it isn't.
Are we ready?	Yes, we are.	No, we aren't.
Are you doctors?	Yes, we are.	No, we aren't.
Are they rich?	Yes, they are.	No, they aren't.

PRONOUNS: SUBJECT, OBJECT, POSSESSIVE

SUBJECT PRONOUN	OBJECT PRONOUN	POSSESSIVE ADJECTIVE
I	me	my
you	you	your
he	him	his
she	her	her
it	it	its
we	us	our
they	them	their

Subject pronouns and object pronouns

SUBJECT PRONOUN		OBJECT PRONOUN	
I	I need help.	ME	Can you help **me**?
YOU	**You** need help.	YOU	Can I help **you**?
HE	**He** needs help.	HIM	Can you help **him**?
SHE	**She** needs help.	HER	Can you help **her**?
IT	**It** needs help.	IT	Can you help **it**?
WE	**We** need help.	US	Can you help **us**?
THEY	**They** need help.	THEM	Can you help **them**?

- I will go to Switzerland this Easter.
- He is being an absolute brat.
- We will meet you all at 20:00h.
- They love mashed potato.

- My boss likes you and will keep you on board.
- I like him. Harry is a really nice guy
- Gemma is a lovely person. My colleagues will love her.
- Can we get this new television? Yeah, let's get it.
- Do they find us to be good partners?

POSSESSIVE ADJECTIVES

POSSESSIVE ADJECTIVE	
MY	**My** shirt is green.
YOUR	**Your** book is new.
HIS	**His** pillow is soft.
HER	**Her** dog is small.
ITS	**Its** bone is old.
OUR	**Our** bird is noisy.
YOUR	**Your** house is big.
THEIR	**Their** car is slow.

- I do really enjoy spending my time with you.
- Your birthday is coming up, what would you like?
- His name is Jack.

THERE IS / ARE

	THERE IS Singular / Uncountable	THERE ARE Plural
Positive	There is ... There's ...	There are ...
Negative	There is not ... There isn't ...	There are not ... There aren't ...
Interrogative	Is there ...?	Are there ...?

Positive Sentences

We use there is for singular and there are for plural.

There is one table in the classroom.

There are three chairs in the classroom.

There is some sugar on the table.

There is ice cream on your shirt.

We also use There is with uncountable nouns:

There is milk in the fridge.

There is some sugar on the table.

There is ice cream on your shirt.

Negative Form

There is not a horse in the field.

There are not eight children in the school.

There is not a tree in the garden.

Questions

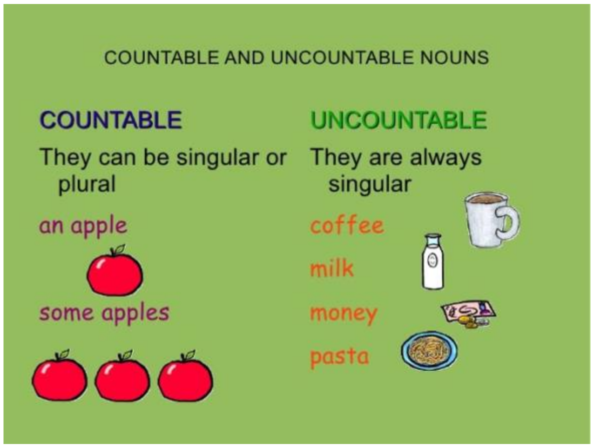
Is there a dog in the supermarket? - No, there isn't.
Are there any dogs in the park? - Yes, there are.
Is there a security guard in the shop? - Yes, there is.

How Many with Are There

How many + plural noun + are there (+ complement).

How many dogs are there in the park?
How many students are there in your class?

COUNTABLE / UNCOUNTABLE NOUNS



Nouns can be countable or uncountable. Countable nouns can be counted, e.g. *an apple*, *two apples*, *three apples*, etc. Uncountable nouns cannot be counted, e.g. *air*, *rice*, *water*, etc. When you learn a new noun, you should check if it is countable or uncountable and note how it is used in a sentence.

Countable nouns

For positive sentences we can use *a/an* for singular nouns or *some* for plurals.

There's ***a*** *man at the door.*

I have ***some*** *friends in New York.*

For negatives we can use *a/an* for singular nouns or *any* for plurals.

I don't have ***a*** *dog.*

There aren't ***any*** *seats.*

Uncountable nouns

Here are some examples of uncountable nouns:

Bread	sugar	salt
Milk	sand	butter

We use *some* with uncountable nouns in positive sentences and *any* with negatives.

*There's **some** milk in the fridge.*

*There isn't **any** coffee.*

Questions

In questions we use *a/an*, *any* or *how many* with countable nouns.

*Is there **an** email address to write to?*

*Are there **any** chairs?*

***How many** chairs are there?*

And we use *any* or *how much* with uncountable nouns.

*Is there **any** sugar?*

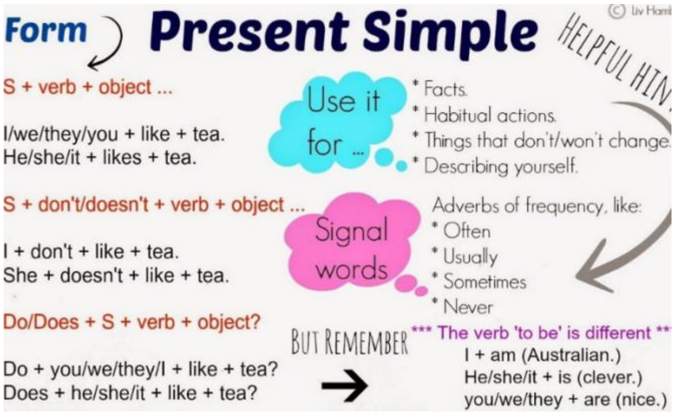
***How much** orange juice is there?*

A lot of (or *lots of*) can be used with both countable and uncountable nouns.

*There are **lots of** apples on the trees.*

*There is **a lot of** snow on the road.*

PRESENT SIMPLE



The simple present tense in English is used to describe an action that is regular, true or normal.

We use the present tense:

1. For repeated or regular actions in the present time period.

- I **take** the train to the office.
- The train to Berlin **leaves** every hour.
- John **sleeps** eight hours every night during the week.

2. For facts.

- The President of The USA **lives** in The White House.
- A dog **has** four legs.
- We **come** from Switzerland.

3. For habits.

- I **get up** early every day.
- Carol **brushes** her teeth twice a day.
- They **travel** to their country house every weekend.

4. For things that are always / generally true.

- It **rains** a lot in winter.
- The Queen of England **lives** in Buckingham Palace.
- They **speak** English at work.

The spelling for the verb in the third person differs depending on the ending of that verb:

1. For verbs that end in **-O, -CH, -SH, -SS, -X,** or **-Z** we add **-ES** in the third person.

- go – goes
- catch – catches
- wash – washes
- kiss – kisses
- fix – fixes
- buzz – buzzes

2. For verbs that end in a **consonant + Y**, we remove the **Y** and add **-IES**.

- marry – marries
- study – studies
- carry – carries
- worry – worries

NOTE: For verbs that end in a **vowel + Y**, we just add **-S**.

- play – plays
- enjoy – enjoys
- say – says

Negative Sentences in the Simple Present Tense

- Affirmative: You speak French.

Negative: You **don't** speak French.

- Affirmative: He speaks German.

Negative: He **doesn't** speak German.

Questions in the Simple Present Tense

- Affirmative: You speak English.

Question: **Do** you speak English?

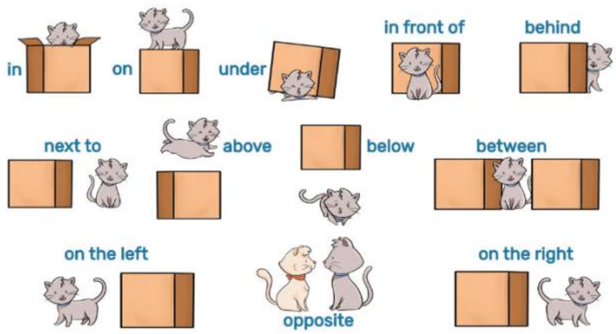
- Affirmative: He speaks French.

Question: **Does** he speak French?

- **Do** you need a dictionary?
- **Does** Mary need a dictionary?
- **Do** we have a meeting now?
- **Does** it rain a lot in winter?
- **Do** they want to go to the party?
- **Does** he like pizza?

Sample Questions	Short Answer (Affirmative)	Short Answer (Negative)
Do you like chocolate?	Yes, I do.	No, I don't.
Do I need a pencil?	Yes, you do.	No, you don't.
Do you both like chocolate?	Yes, we do.	No, we don't.
Do they like chocolate?	Yes, they do.	No, they don't.
Does he like chocolate?	Yes, he does.	No, he doesn't.
Does she like chocolate?	Yes, she does.	No, she doesn't.
Does it have four wheels?	Yes, it does.	No, it doesn't.

PREPOSITIONS OF PLACE



There is a cup on the table.

The helicopter hovered above the house.

The police placed a sheet over the body.

He stood in front of the door and rang the bell.

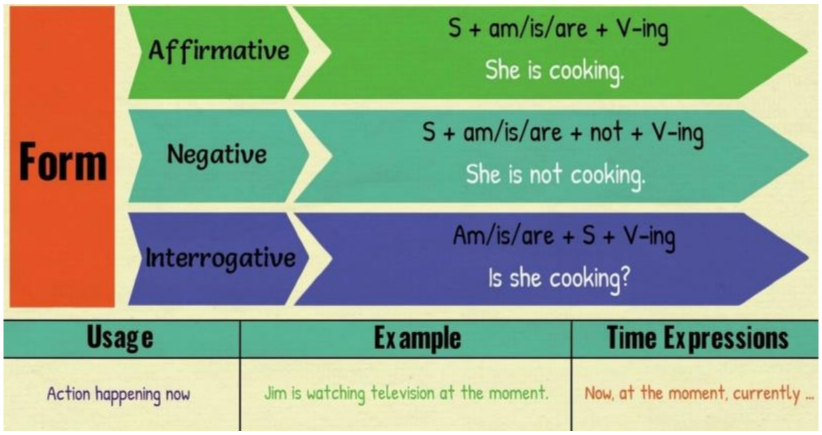
Ram sat beside Tara.

A small stream runs below that bridge.

He put the key under the doormat.

He put his hands behind his back.

PRESENT CONTINUOUS



We use the present progressive tense:

1. When somebody is doing something at this moment.

- Sarah is changing her clothes right now.
- Her boyfriend is waiting for her.
- We are learning the progressive tense in English.

2. When something is happening at this moment. When the action has started but hasn't finished.

- It is snowing at the moment.
- The economy is growing at an exponential rate.
- The children are sleeping so please be quiet.

3. To talk about something that is happening around the time of speaking but not necessarily at that exact moment.

- Alfredo is studying a lot for his exam.
- I'm reading a great book. (Not necessary right at this moment)
- We are planning a trip to Jamaica.

SIMPLE PAST TENSE

Affirmative			Negative			
For all the subject pronouns the form of the verb is the same.			In the negative form, we use an auxiliary and the main verb is in the simple form.			
			Long form		Short form	
Regular verb	I, you, he, she, it, we, you, they	worked	I, you, he, she, it, we, you, they	did not work	I, you, he, she, it, we, you, they	didn´t work
Irregular verb		ate		did not eat		didn´t eat

Interrogative				Short answers		
Regular verb	Did	I, you, he, she, it, we, you, they	work ...?	Yes,	I, you, he, she, it, we, you, they	did.
Irregular verb			eat ...?	No,		didn't.

AFFIRMATIVE:

subject + verb (in past form) + complement.

Example:

I saw a movie yerterday.

NEGATIVE:

subject + auxiliary verb (did) + negation
+ verb(infinitive) + complement.

Example:

He didn't hear the telephone.

QUESTIONS:

Auxiliary verb (did) + subject + verb(infinitive) + complement + ?.

Example:

Did you have dinner last night?

PAST TENSE CONTINUOUS

FORM

Positive

I She He It	was	working.
You We They	were	

Negative

I She He It	wasn't (was not)	working.
You We They	(were not)	

Question

Was	I she he it	working?
Were	you we they	

Short answers

Yes, No,	I she he it	was. wasn't.
	you we they	were. weren't.

It refers to a continuing action or state that was happening at some point in the past. The past continuous tense is formed by combining the past tense of to be (i.e., was/were) with the verb's present participle (-ing word).

The sun was shining every day that summer.

As I spoke, the children were laughing at my cleverness.

It was snowing yesterday.

They were eating at the restaurant.

You were working yesterday.

I was studying last night.

I was waiting for the cab when I met Raj.

The children were shouting when the teacher came in.

It was midnight when it was raining.

Everyone was clapping.

PRESENT PERFECT

The Present Perfect Tense is formed using the following structure:

Affirmative: Subject + Have / Has + Past Participle

Negative: Subject + Haven't / Hasn't + Past Participle

Question: Have / Has + Subject + Past Participle

When do we use the Present Perfect Tense?

1. Unspecified point in the past

- I **have been** to Spain three times.

(At some unspecified time in the past, I went to Spain).

Compare with the simple past:

- I went to Spain three times in 2005.

(specified time in the past - the year 2005)

2. An action that occurred in the past, but has a result in the present (now)

- We can't find our luggage. **Have** you **seen** it?

(The luggage was lost in the past, do you know where it is now?)

3. Talking about general experiences (ever, never)

It usually refers to an event happening at some moment in your life.

- **Has** she ever **tried** Chilean wine before? (in her life)
- I**'ve** never **eaten** monkey brains before. (in my life)

4. Events that recently occurred (just)

- Do you want to go to a restaurant with me?

No, thanks. **I've** just **eaten** lunch. (I recently ate lunch.)

5. Events that have not occurred up to now (yet)

- Are Carlos and Rodrigo here? No, they **haven't arrived** yet. (they're still not here now)

6. Events that occurred before you expected (already)

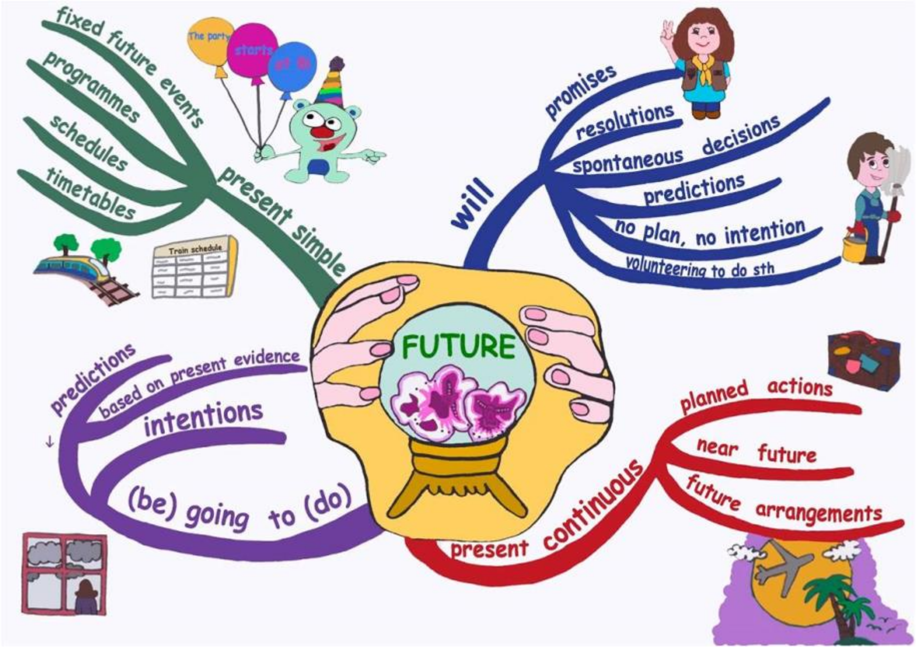
- I**'ve** already **graduated** from University. (I expected to graduate at a later date.)

7. Events that began in the past and haven't changed (for, since)

- Mike **has worked** at Woodward for 3 years.

- Julie **has worked** at Woodward since September last year.

FUTURE FORMS



Use:

Future I going to	Future I Will	Simple Present	Present Progressive
decision made for the future	action in the future that cannot be influenced	action set by a time table or schedule	action already arranged for the near future
conclusion with regard to the future	assumption with regard to the future		
	spontaneous decisión		

We have a lesson next Monday. The train arrives at 6.30 in the morning.

I'm playing football tomorrow. ...

It will be a nice day tomorrow. ...

I hope you will come to my party. ...

I'll see you tomorrow. ...

Tim will be at the meeting. ...

I'm going to drive to work today. ...

READING SECTION

READING 1 MATHEMATICAL LOGIC

Logic means reasoning. The reasoning may be a legal opinion or mathematical confirmation. We apply certain logic in Mathematics. Basic Mathematical logics are a negation, conjunction, and disjunction. The symbolic form of mathematical logic is, '~' for negation '^' for conjunction and ' v ' for disjunction. In this article, we will discuss the basic Mathematical logic with the truth table and examples.

Mathematical Logics Classification

Mathematical logic is classified into four subfields. They are:

- Set Theory
- Model Theory
- Recursion Theory
- Proof Theory

Basic Mathematical Logical Operators

The three logical operators used in Mathematics are:

- Conjunction (AND)
- Disjunction (OR)
- Negation (NOT)

Mathematical Logic Formulas

Conjunction (AND)

We can join two statements by "**AND**" operand. It is also known as a conjunction. Its symbolic form is "∧". In this operator, if anyone of the statement is false, then the result will be false. If both the statements are true, then the result will be true. It has two or more inputs but only one output.

Truth Table for Conjunction (AND)

Input	**Input**	**Output**
A	**B**	**A AND B (A ∧ B)**
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

We can join two statements by "OR" operand. It is also known as disjunction. It's symbolic form is "v". In this operator, if anyone of the statement is true, then the result is true. If both the statements are false, then the result will be false. It has two or more inputs but only one output.

Truth Table for Disjunction (OR)

Input	Input	Output
A	**B**	**A OR B (A ∨ B)**
T	T	T
T	F	T
F	T	T
F	F	F

Negation (NOT)

Negation is an operator which gives the opposite statement of the given statement. It is also known as **NOT**, denoted by **"~"**. It is an operation that gives the opposite result. If the input is true, then the output will be false. If the input is false, then the output will be true. It has one input and one output. The truth table for NOT is given below:

Input	**Output**
A	**Negation A (~A)**
T	F
F	T

Mathematical Logics problems

Example 1:

Write the truth table values of conjunction for the given two statements

A: x is an even number

B: x is a prime number

Solution:

Given: A: x is an even number

B: x is a prime number

Let assume the different x values to prove the conjunction truth table

X value	A	B	A AND B ($A \wedge B$)
X = 2	T	T	T
X = 4	T	F	F
X = 3	F	T	F
X = 9	F	F	F

Example 2:

Write the truth table values of disjunction for the given two statements

A: p is divisible by 2

B: p is divisible by 3

Solution:

Given: A: P is divisible by 2

B: P is divisible by 3

Let assume the different x values to prove the disjunction truth table

Value of P	**A**	**B**	**A OR B (A ∨ B)**
P = 12	T	T	T
P = 4	T	F	T
P = 9	F	T	T
P = 7	F	F	F

Example 3:

Find the negation of the given statement “ a number 6 is an even number”

Solution:

Let “S” be the given statement

S = 6 is an even number

Therefore, the negation of the given statement is

~S = 6 is not an even number.

Therefore, the negation of the statement is “ 6 is not an even number”

READING 2

FUNDAMENTALS OF LOGIC: SYNTAX, SEMANTICS, AND PROOF

Syntax

The syntax of a logic is a detailing of the allowed symbols of a language and the conditions for their grammatical use. It's a set of characters, along with a function that defines the grammar of the language by taking in (finite, in most logics) strings of these characters and returning true or false. This function is usually inductively defined. So for instance in propositional logic we have some set of characters designated to be our propositional variables (we'll denote them p, q, r, …), and another set of characters designated to be our logical symbols (∧, ∨, ¬, →, and parentheses). We inductively define our grammar function G as follows: Every propositional variable is grammatical. If X and Y are grammatical strings, then so are (¬X), (X ∧ Y), (X ∨ Y), and (X → Y). And no other string is grammatical. This definition is what allows us to say that (p ∧ (¬(q ∨ r))) is grammatical but (p ∧ (¬(q ∨ ¬))) is not. The grammar for propositional logic is especially nice and simple, and first-order logic is only mildly more complicated. The important thing is that the syntax for these logics as well as higher order logics is algorithmically checkable; it's possible to write a simple program that verifies whether any given input string counts as grammatical or not.

Semantics

The semantics of a logic is a system for assigning some sort of meaning to the grammatical strings of the language. There are different types of meanings that might be assigned, but the primary one for classical logic is truth and falsity. For propositional logic, our semantics is a set of truth functions, which are functions that take in grammatical strings and return either true or false (you've encountered these before if you've seen truth tables; each row in a truth table corresponds to a particular truth function). Not just any such function will do; these functions will have to satisfy certain constraints, such as that whatever a truth function assigns to a string X, it must assign the opposite value to the string (¬X), and that (A ∧ B) is assigned true if and only if both A and B are assigned true. These constraints on our functions are really what endow our strings with meaning; they induce a sort of structure on the truth values of strings that lines up with our intended interpretation of them.

For propositional logic, we give an inductive definition for our collection of valid truth functions. We construct a truth function by first doing any assignment of truth values to the propositional variables (p, q, r, and so on), and then defining what the function does to the rest of the strings in terms of these truth values. So for any strings X and Y, the truth function assigns true to (¬X) if and only if it assigns false to X. It assigns true to (X ∧ Y) if and only if it assigns true to X and true to Y. And so on. By our construction of the grammar of propositional logic, we've guaranteed that our function is defined on all grammatical strings. Of course, there are many such truth functions (one for every way of assigning truth values to the propositional variables) – and this collection of truth functions is what defines our semantics for propositional logic.

First order logic is again more complicated than propositional logic in its semantics, but not hugely so. When talking about the semantics of first-order logic, we call its truth functions models (or structures), and each model comes with a universe of objects that the quantifiers of the logic range over. An important difference between the semantics of propositional logic and the semantics of first order logic is that in the former, we can construct an algorithm that directly searches through the possible truth functions. In the latter, getting a similar sort of direct access to the models is much more difficult. Consider, for instance, that some models will have universes with infinitely many elements, meaning that to verify the truth of a universally quantified statement requires verifying it for infinitely many elements.

Last thing I need to say about semantics before moving on to proof: there is the all-important concept of semantic entailment. A set of sentences (call it A) is said to semantically entail another sentence (call it X), if every truth function that assigns everything in A true also assigns X true. When this is the case, we write: $A \models X$.

Examples

$\{(p \wedge q)\} \models p$

$\{(p \vee q), (\neg q)\} \models p$

$\{p, (p \to q), (q \to r)\} \models r$

READING 3

FORMAL LOGIC AND INFORMAL LOGIC

Douglas Walton: Formal logic has to do with the forms of argument (syntax) and truth values (semantics). . . . Informal logic (or more broadly argumentation)), as a field, has to do with the uses of argumentation in a context of dialogue, an essentially pragmatic undertaking. Hence the strongly opposed current distinction between informal and formal logic is really an illusion, to a great extent. It is better to distinguish between the syntactic/semantic study of reasoning, on the one hand, and the pragmatic study of reasoning in arguments on the other hand. The two studies, if they are to be useful to serve the primary goal of logic, should be regarded as inherently interdependent, and not opposed, as the current conventional wisdom seems to have it.

Dale Jacquette: Formal logicians of a radical stripe often dismiss informal logical techniques as insufficiently rigorous, precise, or general in scope, while their equally vehement counterparts in the informal logic camp typically regard algebraic logic and set theoretical semantics as nothing more than an empty formalism lacking both theoretical significance and practical application when not informed by the informal logical content that formal logicians pretend to despise.

READING 4

What is Logic?

Logic is the basis of all mathematical reasoning, and of all automated reasoning. The rules of logic specify the meaning of mathematical statements. These rules help us understand and reason with statements such as –

\exists x such that x\neq a^2 + b^2, where \:x,a,b\in Z

Which in Simple English means "There exists an integer that is not the sum of two squares". Importance of Mathematical Logic The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments. Apart from its importance in understanding mathematical reasoning, logic has numerous applications in Computer Science, varying from design of digital circuits, to the construction of computer programs and verification of correctness of programs.

Propositional Logic

What is a proposition? A proposition is the basic building block of logic. It is defined as a declarative sentence that is either True or False, but not both. The Truth Value of a proposition is True(denoted as T) if it is a true statement, and False(denoted as F) if it is a false statement.

For Example,

1. The sun rises in the East and sets in the West.

2. 1 + 1 = 2

3. 'b' is a vowel.

All of the above sentences are propositions, where the first two are Valid(True) and the third one is Invalid(False). Some sentences that do not have a truth value or may have more than one truth value are not propositions.

For Example,

1. What time is it?

2. Go out and play.

3. $x + 1 = 2$.

The above sentences are not propositions as the first two do not have a truth value, and the third one may be true or false. To represent propositions, propositional variables are used. By Convention, these variables are represented by small alphabets such as p,\:q,\:r,\:s . The area of logic which deals with propositions is called propositional calculus or propositional logic. It also includes producing new propositions using existing ones. Propositions constructed using one or more propositions are called compound propositions. The propositions are combined together using Logical Connectives or Logical Operators.

Truth Table

Since we need to know the truth value of a proposition in all possible scenarios, we consider all the possible combinations of the propositions which are joined together by Logical Connectives to form the given compound proposition. This compilation of all possible scenarios in a tabular format is called a truth table. Most Common Logical Connectives-

1. Negation – If p is a proposition, then the negation of p is denoted by \neg p , which when translated to simple English means- "It is not the case that p " or simply "not p ". The truth value of \neg p is the opposite of the truth value of p . The truth table of \neg p is-

\begin{tabular}{ |c|c| } \hline p & \neg p \\ \hline \hline T & F \\ \hline F & T \\ \hline \end{tabular}

Example, The negation of "It is raining today", is "It is not the case that is raining today" or simply "It is not raining today".

2. Conjunction – For any two propositions p and q , their conjunction is denoted by p\wedge q , which means "p and q ". The conjunction p\wedge q is True when both p and q are True, otherwise False. The truth table of p\wedge q is-

p	q	$p\wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example, The conjunction of the propositions p – "Today is Friday" and q – "It is raining today", p\wedge q is "Today is Friday and it is raining today". This proposition is true only on rainy Fridays and is false on any other rainy day or on Fridays when it does not rain.

3. Disjunction – For any two propositions p and q , their disjunction is denoted by p\vee q , which means "p or q ". The disjunction p\vee q is True when either p or q is True, otherwise False. The truth table of p\vee q is-

p	q	$p\vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example, The disjunction of the propositions p – "Today is Friday" and q – "It is raining today", p\vee q is "Today is Friday or it is raining today". This proposition is true on any day that is a

Friday or a rainy day(including rainy Fridays) and is false on any day other than Friday when it also does not rain.

4. Exclusive Or – For any two propositions p and q, their exclusive or is denoted by $p\oplus q$, which means "either p or q but not both". The exclusive or $p\oplus q$ is True when either p or q is True, and False when both are true or both are false. The truth table of $p\oplus q$ is-

p	q	$p\oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Example, The exclusive or of the propositions p – "Today is Friday" and q – "It is raining today", $p\oplus q$ is "Either today is Friday or it is raining today, but not both". This proposition is true on any day that is a Friday or a rainy day(not including rainy Fridays) and is false on any day other than Friday when it does not rain or rainy Fridays.

5. Implication – For any two propositions p and q, the statement "if p then q" is called an implication and it is denoted by $p\rightarrow q$. In the implication $p\rightarrow q$, p is called the hypothesis or antecedent or premise and q is called the conclusion or consequence. The implication is $p\rightarrow q$ is also called a conditional statement. The implication is false when p is true and q is false otherwise it is true. The truth table of $p\rightarrow q$ is-

p	q	$p\rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

You might wonder that why is $p\rightarrow q$ true when p is false. This is because the implication guarantees that when p and q are true then the implication is true. But the implication does not guarantee anything when the premise p is false. There is no way of knowing whether or not the implication is false since p did not happen. This situation is similar to the “Innocent until proven Guilty” stance, which means that the implication $p\rightarrow q$ is considered true until proven false. Since we cannot call the implication $p\rightarrow q$ false when p is false, our only alternative is to call it true. This follows from the Explosion Principle which says- “A False statement implies anything” Condi

tional statements play a very important role in mathematical reasoning, thus a variety of terminology is used to express $p\rightarrow q$, some of which are listed below.

"if , then "" is sufficient for "" when ""a necessary condition for is "" only if ""
unless "" follows from "

Example, “If it is Friday then it is raining today” is a proposition which is of the form . The above proposition is true if it is not Friday(premise is false) or if it is Friday and it is raining, and it is false when it is Friday but it is not raining.

6. Biconditional or Double Implication –

For any two propositions p and q , the statement “p if and only if(iff) q ” is called a biconditional and it is denoted by $p\leftrightarrow q$. The statement $p\leftrightarrow q$ is also called a bi-implication. $p\leftrightarrow q$ has the same truth value as $(p\rightarrow q) \wedge (q\rightarrow p)$ The implication is true when p and q have same truth values, and is false otherwise. The truth table of $p\leftrightarrow q$ is-

p	q	$p\leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Some other common ways of expressing $p\leftrightarrow q$ are-

"p is necessary and sufficient for q""if p then q, and conversely""p if q"

Example, “It is raining today if and only if it is Friday today.” is a proposition which is of the form $p\leftrightarrow q$. The above proposition is true if it is not Friday and it is not raining or if it is Friday and it is raining, and it is false when it is not Friday or it is not raining. Exercise:

1) Consider the following statements:

P: Good mobile phones are not cheap.

Q: Cheap mobile phones are not good.

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT?

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE.

READING 5

PROPOSITIONS

English sentences are either true or false or neither. Consider the following sentences:

1. Warsaw is the capital of Poland.

2. 2+5=3.

3. How are you?

The first sentence is true, the second is false, while the last one is neither true nor false. A statement

that is either true or false but not both is called a proposition. Propositional logic deals with such

statements and compound propositions that combine together simple propositions (e.g., combining

sentences (1) and (2) above we may say “Warsaw is the capital of Poland and 2+5=3”).

In order to build compound propositions we need rules on how to combine propositions. We

denote propositions by lowercase letters p, q or r. Let us define: The conjunction of p and q, denoted as p ^ q, is the proposition

p and q;

and it is true when both p and q are true and false otherwise. The disjunction of p and q, denoted as p _ q, is the proposition

p or q;

and it is false when both p and q are false and true otherwise. The negation of p, denoted either as :p or p, is the proposition

It is not true that p.

Example 1: Let p =“Hawks swoop” and q =“Gulls glide”. Then p _ q is the same as “Hawks swoop

or gulls glide”. We also can translate back. For example, the English sentence “it is not true that hawks swoop" can be written as :p.

Exercise 1A: With the same notation as in the example above write the following propositions symbolically: It is not true that "Hawks swoop and gulls glide". "Hawks do not swoop or gulls do not glide".

READING 6

TRUTH TABLES

We can express compound propositions using a truth table that displays the relationships between the truth values of the simple propositions and the compound proposition. In the next three tables we show the truth tables for the negation, conjunction, and disjunction. Observe that any proposition p can take only two values, namely true, denoted T , or false, denoted F . Therefore, for a compound proposition consisting of two propositions (e.g., p ^ q) we must consider only four possible assignments of T and F .

Table 1: The truth table for the negation.

p	$\neg p$
T	F
F	T

Table 2: The truth table for the conjunction.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 3: The truth table for the disjunction.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

To construct a truth table for a statement (e.g., :p _ q) containing two propositions, say p and q, one first builds two columns with all possible vales of p and q (i.e., (T;T); (T;F); (F; T); (F; F)), and then follows already accepted rules of inference to determine the truth value of the compound statement (say :p _ q).

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